On Multitasking and Job Design in Relational Contracts*

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Abstract

We investigate the optimal job design in a repeated principal-agent relationship with multiple tasks where the performance measurement is distorted, aggregated, and nonverifiable. We compare task bundling, where all the tasks are assigned to a single agent, with task separation, where the tasks are split and assigned to two agents. Compared to task bundling, task separation mitigates misallocation of effort among the tasks but tightens the self-enforcing constraint due to dispersion of informal bonuses to multiple agents. Consequently, task separation is better than task bundling if and only if the discount factor of the parties is high. We also consider an extended model in which the principal combines explicit incentive pays based on a verifiable and distorted signal. In such cases, task separation is optimal if the discount factor is sufficiently high or sufficiently low.

1 Introduction

In complex work places, uncertain contingencies and states are usually hard to be described *ex ante* and/or verified *ex post*. As a result, it is impossible to specify comprehensive working and incentive rules *ex ante* through formal contracts. Management practices supporting high productivity are rather based on relational contracts (Gibbons and Henderson, 2013). Long-term relationship allows the parties to support informal rules that cannot be specified through formal contracts, which often contributes to substantial improvement of organizational performance.

The difficulty of specification of working rules arises especially when there are multiple tasks involved in the working place. A notable example of multiple tasks with implicit incentives include product innovation. Successful product innovation

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usually consists of multiple steps of the process, in each of which it is difficult to exactly specify activities or effort levels *ex ante* in a formal contract. Interestingly, a part of the tasks in the process is often outsourced to outside of the firm. For instance, pharmaceutical firms usually form cross-functional product teams for developing a new drug. Azoulay (2004) and Azoulay et al. (2010) report that clinical trials, which are necessary processes for drug development, was outsourced in some firms while it was also implemented within the team in other firms. A similar outsourcing pattern is observed in fund management. Mutual fund managers manage multiple funds, each of which requires them non-contractible tasks such as to find investment opportunities and to assess the risk and return. Although to manage multiple funds simultaneously is recently widespread, mutual fund firms sometimes delegate management of a fund to another firm (Chen et al., 2013; Agarwal et al., 2015).

Such outsourcing decisions are closely related to job design problems in the organizations. The outsourced tasks are assigned to a different worker/division. If a part of the tasks are away from a worker with multiple tasks, then she can at least partly mitigate the multitasking problem that causes a tension among the assigned tasks. At the same time, splitting tasks in such a way induces the firm to manage the extra worker/division in addition to the existing one. Therefore task assignment has potentially positive and negative impacts on the workers incentive for multiple activities through change of the set of the worker's choice and of the way to evaluate them.

The purpose of this article is to clarify effects of job design in such complex working environments relying on relational contracts. As our baseline model, we consider a repeated moral hazard model with multiple tasks in which the performance measurement is aggregated, distorted, and nonverifiable. Nonverifiability of the performance measurement implies that an informal agreement supported by a relational contract is indispensable for incentive provision. In addition to designing a relational contract, the organization chooses a mode of job design: either *task bundling*, under which a single agent performs all the tasks; or *task separation*, under which the tasks are split and assigned to two agents.

The baseline model clarifies an intuitive trade-off between task bundling and task separation. When the aggregated performance measurement is distorted, the allocation of effort made by an agent must also be distorted from the principal's desirable level. Under task separation, on the one hand, each agent performs fewer tasks than task bundling, which mitigates the misallocation of effort. On the other hand, the principal must deal with discretionary bonuses to multiple agents. The dispersion of discretionary bonuses under task separation makes it harder to sustain relational contracts. In other words, relative to task separation, task bundling has an advantage in terms of self-enforcement. Consequently, we obtain a clear-cut relationship between the modes of job design: task separation is preferable if and only if the parties are patient.

The optimal job design also depends on how serious the multitasking problem is. Under task separation, the principal is able to discriminate bonuses between the agents, which mitigates misallocation of effort caused by the multitasking problem. If the multitasking problem is relatively innocuous so that the principal does not have to make a bonus difference between the agents, then task bundling does not cause misallocation of effort seriously and tends to be preferred. By contrast, if the principal desires to discriminate bonuses largely under task separation, then task bundling substantially boosts distortion of effort allocation and tends to be avoided. Furthermore, if the difference of the desired bonuses is extremely large under task separation, then task separation may exhibit *task exclusion*, under which one of the agents does not perform the tasks at all. Note that task exclusion may also interpreted as task bundling with restriction to a limited number of tasks to be performed. Exclusion of unimportant tasks can mitigate distortion of effort allocation without providing additional incentives.

In real-life management, informal bonuses are often combined with explicit incentive schemes based on objective performance measurements. In order to consider such cases, we extend our baseline model such that, in addition to the nonverifiable performance measurement, there is a verifiable performance measurement that is also distorted. The principal may design explicit incentives through a short-term formal contract as well as implicit incentives. In such environments, relative to task bundling, task separation mitigates misallocation of effort through explicit incentive schemes as well. This benefit unambiguously makes task separation better than task bundling when the discount factor is so low that no informal bonuses are promised. As a result, task bundling is strictly optimal only when the discount factor is in an intermediate range. We demonstrate that if to separate the tasks substantially resolves the multitasking problem only through the explicit incentives, then task separation is strictly preferred for any discount factor.

The present article relates to a broad strand of theoretical literature on moral hazard and job design. As in the present article, the analysis of job design is often motivated by multitasking problems that cause misallocation of effort due to distortion of the performance measurements.[1] The existing literature discusses a trade-off between task bundling and task separation, which provides a richer insight on organization design. On the one hand, mitigation of misallocation of effort is usually a bright side of task separation. On the other hand, dark sides of task separation include: additional risk premium payments in case of risk-averse agents (Itoh, 1994, 2001; Besanko et al., 2005; Corts, 2007); or excess rent provision due to limited liability (Kragl and Schöttner, 2014).[2] All of these articles analyze a one-shot moral hazard model with multiple tasks and verifiable performance measurements. Our analysis is different from them in that we consider a repeated and relational incentive contract and find a new insight on job design. Especially, we first point out dispersion of informal bonuses as a cost of task separation when incentive provision must be relational.

The present article is also positioned in the literature of relational incentive contracts and organizational design since Baker et al. (2002) and Levin (2003).[3] Recently, there are a number of articles which study relational contracting in a multitasking environment (Schmidt and Schnitzer, 1995; Daido, 2006; Kvaløy and Olsen, 2008; Schöttner, 2008; Mukherjee and Vasconcelos, 2011; Ishihara, 2016). Among them, the most related article to our analysis is Schöttner (2008), who considers a setup similar to our model. She highlights a trade-off between mitigating misallocation of effort under task separation and strengthening punishment for reneging under task bundling. Consequently, she argues that task bundling is preferred if and only if the parties are patient, which is totally opposite to our argument. These opposing arguments are due to different assumptions of observability and verifiability of performance measurements. Especially, she focuses on cases in which the true performance is nonverifiable but observable to the parties, which is interpreted as a special case of our model. As we demonstrate later in Section 5, task separation is rather generically optimal for patient parties as long as the nonverifiable performance measurement is not perfectly correlated to the true performance. Mukherjee and Vasconcelos (2011) consider a different relational contracting model with job design, but argue a trade-off similar to our result. Specifically, in their model, the number of working agents and assigned tasks for each agent are fixed and the modes of job design is either: individual assignment such that the discretionary bonus is contingent only on a single performance measurement; or team

[1] For issues of multitasking problems, see, for instance, Holmström and Milgrom (1991) and Baker (2002).

[2] Kragl and Schöttner (2014) also demonstrate the optimality of task exclusion as in our anlysis.

[3] See also Malcomson (2013) for a recent survey.

assignment such that the discretionary bonus is contingent on multiple performance measurements.[4] Individual and team assignment in their model play a similar role to task bundling and separation, respectively, in our model. Specifically, they demonstrate that individual assignment has an advantage in terms of self-enforcement while team assignment can mitigate misallocation of effort. Furthermore, team assignment is optimal for high discount factor and low discount factor when one of the performance measurements is verifiable. Although we share a part of the economic insights with them, our multitasking environments has a different structure of job design.

The rest of the article is organized as follows. The next section describes our baseline model. Section 3 derives an optimal contract under each job design and section 4 compares the optimal contract of each job design to derive the pattern of the optimal job design. Section 5 extends our model by bringing a verifiable performance measurement into the baseline model. The final section concludes. The Appendix contains all the proofs and supplementary analyses.

2 The Model

2.1 Baseline Setup

To clarify the main idea in this article intuitively, we first consider a baseline setup in which there is no verifiable performance measurement.

There are three players: principal and agents 1 and 2, all of whom are risk-neutral and live in $t = 0, 1, \dots$ until infinity with discount factor $\delta \in (0, 1)$. There are N productive tasks, denoted by $n = 1, \dots, N$. Before period 0, the principal first chooses job design, either *task bundling* or *task separation*, which is fixed for all the subsequent periods.[5] Let $\mathcal{N} \equiv \{1, \dots, N\}$ be the set of the tasks and $(\mathcal{N}_1, \mathcal{N}_2)$ be a partition of $\mathcal{N}$ satisfying $\mathcal{N}_i \neq \emptyset$ for $i = 1, 2$. In each period t, if the parties engage in a formal contract, then depending on the job design, either agent 1 or 2 chooses an effort level $e_{nt} \in [0, \infty)$. Under task bundling, all the tasks in $\mathcal{N}$ are assigned to agent 1 and he chooses e_{nt} for all $n \in \mathcal{N}$. Under task separation, for each $i = 1, 2$, the tasks in $\mathcal{N}_i$ are assigned to agent i and then agent i chooses e_{nt} for each $n \in \mathcal{N}_i$. Assume that the partition $(\mathcal{N}_1, \mathcal{N}_2)$ is exogenously fixed in all periods, meaning that the parties cannot arrange the task

[4]This setup is based on Corts (2007) and is extended to a relational contracting model.

[5]The related articles (e.g., Schöttner (2008); Mukherjee and Vasconcelos (2011)) adopt the same assumption. Except Section 5, all of our results still holds even if the principal chooses job design in each period.

allocation between the agents under task separation.[6]

Given $\boldsymbol{e} \equiv (e_{1t}, \ldots, e_{Nt})$, the principal stochastically receives private benefit: either 1 with probability $f(\boldsymbol{e}) \equiv \min\{\sum_{n\in\mathcal{N}} \alpha_n e_{nt}, 1\}$; or 0 with probability $1 - f(\boldsymbol{e})$. We assume that the benefit is privately observable to the principal.[7] In addition, the parties observe a stochastic signal, which we call the performance measurement: either $x_t = S$ with probability $p(\boldsymbol{e}) \equiv \min\{\sum_{n\in\mathcal{N}} \mu_n e_{nt}, 1\}$; or $x_t = F$ with probability $1-p(\boldsymbol{e})$. The agent agent who is responsible to task n endures cost $c_n(e_{nt}) \equiv \gamma_n e_{nt}^2/2$ from exerting effort on task n. The total cost of the agent is expressed as an additive separable way. Then agent 1's total cost under task bundling is $\sum_{n\in\mathcal{N}} c_n(e_{nt})$ and agent i's cost under task separation is $\sum_{n\in\mathcal{N}_i} c_n(e_{nt})$.[8] Both the effort level and the amount of the costs are privately observable to the assigned agent.

In each period t, the principal offers a short-term formal contract, which specifies a fixed amount of monetary transfer $w_{it} \in \mathbb{R}$ to agent i. In addition, the parties implicitly promise a discretionary bonus $b_{it} \in \mathbb{R}$ that may be contingent on nonverifiable performance measure x_t. Since the discretionary bonus is informal, the principal can renege after realization of x_t.

As Figure 1 illustrates, the parties play the following game in period t.

1. The principal offers a formal contract (and implicit promises) to the agents.

2. Each agent chooses to accept or reject the contract. If an agent chooses rejection, then he receives the value of the outside option 0 and does not choose effort. If both agents reject the contract, then the principal receives the value of the outside option 0 and the period ends.

3. If an agent accepts the contract, then according to the specified job design, he chooses e_{nt} for each $n \in \mathcal{N}$ and the nonverifiable performance measurement x_t is realized.

4. The principal chooses to honour the discretionary bonus or not.

In the following analysis, we often use several vector expressions. Specifically, let

$$\boldsymbol{\alpha} \equiv (\alpha_1, \ldots, \alpha_N), \qquad \boldsymbol{\mu} \equiv (\mu_1, \ldots, \mu_N),$$

[6] This assumption implies that the principal cannot choose an optimal task allocation to each agent under task separation. We briefly discuss this point in Section 6.

[7] We maintain our result as long as the principal's expected benefit is linear in $\boldsymbol{e}$ and she cannot perfectly identify $\boldsymbol{e}$ from observing the private benefit.

[8] Additive separability of the costs means that each task is independent in that the choice of effort on a task does not affect the marginal costs of effort on another task. In Appendix C, we discuss how additive separability may be relaxed.

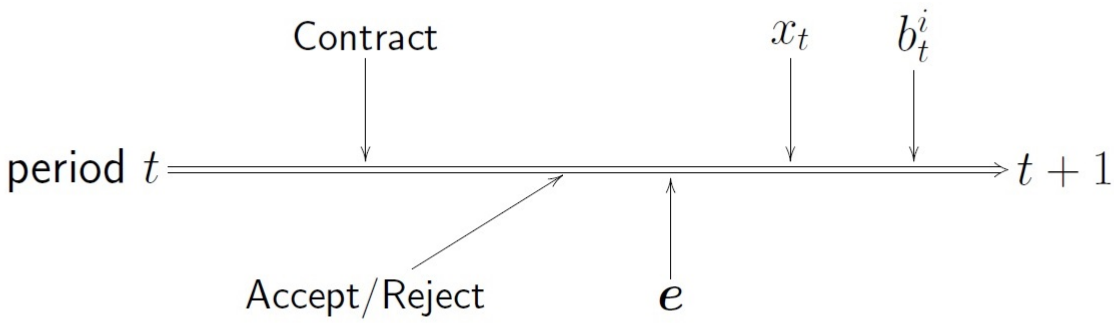

Figure 1: Timing in Period t

$$\boldsymbol{\alpha}_i \equiv (\nu_{1i}\alpha_1, \ldots, \nu_{Ni}\alpha_N), \qquad \boldsymbol{\mu}_i \equiv (\nu_{1i}\mu_1, \ldots, \nu_{Ni}\mu_N),$$

be n-dimensional vectors for $i = 1, 2$, where $\nu_{ni} = \mathbf{1}\{n \in \mathcal{N}_i\}$ is an indicator function that takes 1 if task n is assigned to agent i under task separation. The transpose of a vector is indicated by a superscript prime, e.g., $\boldsymbol{\alpha}'$ and $\boldsymbol{\mu}'$. Furthermore, let $\boldsymbol{\Gamma} \equiv \text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_n)$ be an $n \times n$ diagonal matrix.

Each party's payoff is quasi-linear in monetary transfer. Specifically, given that the agents agree to a formal contract and the principal pays W_{it} to agent i for each $i = 1, 2$, the principal's (expected) payoff in period t is $f(\boldsymbol{e}_t) - \sum_{i=1}^{2} W_{it}$ and each agent's (*ex post*) payoff is $W_{1t} - \sum_{n \in \mathcal{N}} c_n(e_{nt})$ and W_{2t} under task bundling and $W_{1t} - \sum_{n \in \mathcal{N}_1} c_n(e_{nt})$ and $W_{2t} - \sum_{n \in \mathcal{N}_2} c_n(e_{nt})$ under task separation. We assume that for each $n \in \mathcal{N}$, $\alpha_n > 0$, $\mu_n > 0$, and $\gamma_n > 0$. Furthermore, α_n and μ_n are assumed to be sufficiently small relative to γ_n, which guarantees that we may assume $f(\boldsymbol{e}_t) \equiv \sum_{n \in \mathcal{N}} \alpha_n e_{nt}$ and $p(\boldsymbol{e}_t) \equiv \sum_{n \in \mathcal{N}} \mu_n e_{nt}$ throughout the analysis. Let $Y(\boldsymbol{e}_t) \equiv \sum_{n \in \mathcal{N}} [\alpha_n e_{nt} - c_n(e_{nt})]$ be the total surplus shared by the parties. Note that the total surplus is not directly influenced by job design, which means that job design has no technological effect on efficiency. Although we recognize the importance of technological aspects through job design, abstracting the technological effects allows us to clarify the incentive effect of job design.

As in the literature of relational contracting, we characterize a perfect public equilibrium that is optimal for the principal.[9] By applying the analogy in Levin (2003), without loss of generality an optimal equilibrium is stationary in that a pair of fixed transfer (w_1, w_2), a pair of discretionary bonus rule $(b_1(\cdot), b_2(\cdot))$ contingent on x, and effort profile $\boldsymbol{e} \equiv (e_1, \ldots, e_N)$ are time-invariant on the equilibrium path.[10] We define the optimal relational contract as a pair of the fixed transfers, discretionary bonuses,

[9]The formal definition of strategy and equilibrium in this repeated game is in Appendix A.1.

[10]Furthermore, it is also well-known that any Pareto-optimal equilibrium can be implemented by the same stationary equilibrium with modified fixed transfers. The formal proof is omitted.

and effort profile of the optimal stationary public perfect equilibrium. Hereafter drop the time script for each variable since we focus on optimal relational contracts that are stationary.

In Section 3, we characterize an optimal relational contract under each job design, task bundling and task separation. Then, in Section 4, we compare the performance of each job design, by which we characterize the optimal job design.

3 Optimal Contract

3.1 Task Bundling

We first suppose that the principal chooses task bundling before period 0. Under task bundling, no task is assigned to agent 2 and then providing agent 2 with incentives based on the performance measurement does not improve efficiency at all. Thus the principal focuses on the incentive problem of agent 1. Let $\overline{\beta}$ be the discretionary bonus paid to agent 1 when the performance measurement exhibits $x = S$. Then, according to Levin (2003, Theorem 3), the optimal relational contract can be characterized by the following procedure.[11]

Lemma 1 *Suppose that the optimal job design is task bundling. Then the effort profile* $\boldsymbol{e}$ *and discretionary bonus* $\overline{\beta}$ *paid to agent* 1 *for* $x = S$ *of the optimal relational contract solves the following optimization problem:*

$$\max_{\overline{\beta}, \boldsymbol{e}} \quad Y(\boldsymbol{e})$$

$$\text{subject to} \quad \boldsymbol{e} \in \arg\max_{\tilde{\boldsymbol{e}} \in \mathbb{R}_+^N} \left[\overline{\beta} p(\tilde{\boldsymbol{e}}) - \sum_{n \in \mathcal{N}} c_n(\tilde{e}_n) \right], \tag{1}$$

$$-\overline{\beta} + \frac{\delta}{1-\delta} Y(\boldsymbol{e}) \geq 0. \tag{2}$$

Furthermore, the optimized value is the principal's payoff.

Constraints (1) and (2) correspond to the *incentive compatibility constraint* and the *dynamic enforcement constraint*, respectively. The incentive compatibility constraint guarantees that agent 1 voluntarily chooses the targeted effort profile $\boldsymbol{e}$. The dynamic enforcement constraint guarantees that the principal credibly honours the discretionary bonus $\overline{\beta}$ for $x = S$. The left hand side of (2) is the principal's net present value when

[11] The formal proof is omitted.

she keeps the promise: paying $\overline{\beta}$ guarantees the future relationship and generates the optimal continuation payoff. If the promise is reneged on, then the principal pays no informal bonus, which causes termination of the relationship and generates value 0 from the outside option as the continuation payoff.

Provided that $\overline{\beta} \geq 0$, the incentive compatibility constraint can be simplified by the first order condition: for all $n \in \mathcal{N}$,

$$e_n = \overline{\beta}\frac{\mu_n}{\gamma_n}. \tag{3}$$

Then substituting (3) into the objective function and the dynamic enforcement constraint (2) transforms the optimization problem to:

$$\max_{\overline{\beta}\in\mathbb{R}_+} A\overline{\beta} - \frac{M}{2}\overline{\beta}^2 \quad \text{subject to} \quad \frac{1}{r}\left(A\overline{\beta} - \frac{M}{2}\overline{\beta}^2\right) \geq \overline{\beta}, \tag{4}$$

where $r \equiv (1-\delta)/\delta$ is the discount rate, $A \equiv \boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'$, and $M \equiv \boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'$.[12]

The solution to the optimization problem characterizes the optimal relational contract as follows.

Proposition 1 *Suppose that the optimal job design is task bundling. Then the optimal relational contract satisfies the following property.*

1. *For $r < A/2$, the dynamic enforcement constraint is not binding.*
2. *For $r \in [A/2, A]$, the dynamic enforcement constraint is binding and $\overline{\beta} > 0$.*
3. *For $r > A$, $\overline{\beta} = 0$.*
4. *The principal's payoff Y^B is*
   - *(a) non-increasing in $r > 0$; and*
   - *(b) strictly decreasing in $r \in (A, A/2)$.*

As a summary of Proposition 1, Figure 2 illustrates the principal's payoff of the optimal relational contract, denoted by Y^B, where the horizontal axis is the discount rate. It is not surprising to observe that since the discount rate means the degree of impatience of the parties, the principal's payoff is higher as the discount rate is lower. If the parties are sufficiently patient, then the commitment problem is innocuous so

[12]Strictly speaking, the principal can choose negative $\overline{\beta}$, which induces the agent to choose zero effort for all the tasks. However, it is easy to check that this outcome is never strictly optimal and therefore we focus on non-negative $\overline{\beta}$.

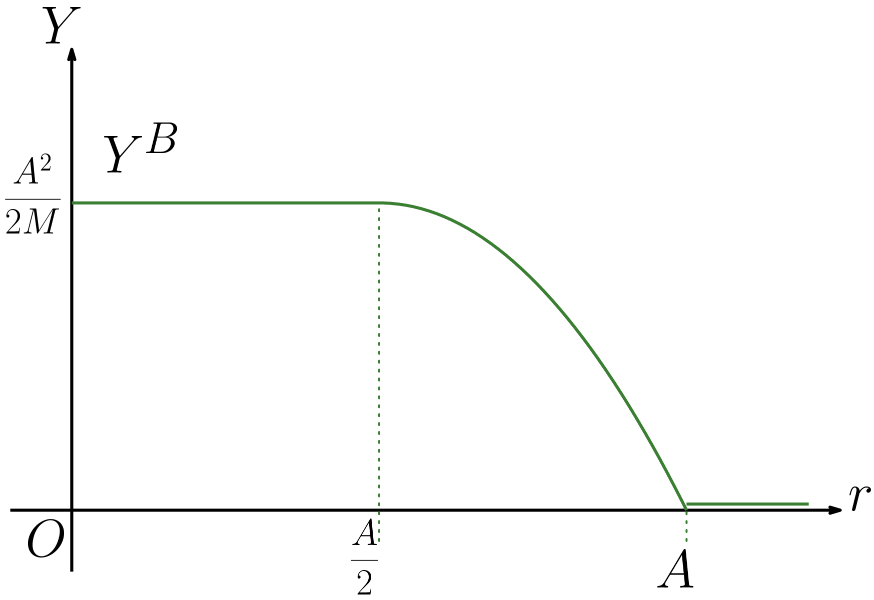

Figure 2: Principal's Payoff under Task Bundling

that the dynamic enforcement constraint is not binding. If the parties' patience is intermediate, then self-enforcement of informal bonuses becomes a concern and the principal's payoff is decreasing in the discount rate. Finally, the sufficiently impatient parties cannot support positive informal bonuses at all and no effort is induced.

3.2 Task Separation

We next suppose that the principal chooses task separation, under which she must deal with the incentive problems of both agents. For each $i = 1, 2$, let $\boldsymbol{e}_i \equiv (e_n)_{n \in \mathcal{N}_i} \in \mathbb{R}_+^{\#\mathcal{N}_i}$ be a $\#\mathcal{N}_i$-dimensional effort vector chosen by agent i and β_i be the discretionary bonus paid to agent i when the performance measurement exhibits $x = S$. Then, similar to task bundling, an optimal relational contract can be summarized by a solution to the following optimization problem.

Lemma 2 *Suppose that the optimal job design is task separation. Then the effort profile $\boldsymbol{e} \equiv (\boldsymbol{e}_1, \boldsymbol{e}_2)$ and the pair of the discretionary bonuses (β_1, β_2) for $x = S$ of the optimal relational contract solves the following optimization problem:*

$$\max_{\beta_1, \beta_2, (\boldsymbol{e}_1, \boldsymbol{e}_2) \in \mathbb{R}_+^N} \quad Y(\boldsymbol{e})$$

$$\textit{subject to} \quad \boldsymbol{e}_i \in \arg\max_{\tilde{\boldsymbol{e}}_i \in \mathbb{R}_+^{\#\mathcal{N}_i}} \left[\beta_i p(\tilde{\boldsymbol{e}}_i, \boldsymbol{e}_{-i}) - \sum_{n \in \mathcal{N}_i} c_n(\tilde{e}_n^i) \right], \ \forall i = 1, 2, \tag{5}$$

$$-\sum_{i=1}^{2}\beta_i + \frac{\delta}{1-\delta}Y(\boldsymbol{e}) \geq 0. \tag{6}$$

Furthermore, the optimized value is the principal's payoff.

Constraints (5) and (6) are again the incentive compatibility constraint and the dynamic enforcement constraint, respectively. Note that the left hand side of the dynamic enforcement constraint captures the sum of the discretionary bonuses over the agents since the principal now must pay the bonuses to both agents.

Similar to task bundling, under task separation, provided that $\beta_i \geq 0$, the incentive compatibility constraint (5) can be simplified by the first order condition: for all $i = 1, 2$ and $n \in \mathcal{N}_i$,

$$e_n = \beta_i \frac{\mu_n}{\gamma_n}. \tag{7}$$

Plugging (7) transforms the optimization problem into:

$$\max_{(\beta_1,\beta_2)\in\mathbb{R}_+^2} \sum_{i=1}^{2}\left(a_i\beta_i - \frac{m_i}{2}\beta_i^2\right) \text{ subject to } \frac{1}{r}\sum_{i=1}^{2}\left(a_i\beta_i - \frac{m_i}{2}\beta_i^2 - \underline{y}\right) \geq \sum_{i=1}^{2}\beta_i, \tag{8}$$

where $a_i \equiv \boldsymbol{\alpha}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}_i'$ and $m_i \equiv \boldsymbol{\mu}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}_i'$.[13] Under task separation, the solution to the problem depends on the new parameters a_i and m_i. Hereafter, we assume without loss of generality a_1 is at least equal to a_2.

Assumption 1 $a_1 \geq a_2$.

Similar to task bundling, the characteristics of the solution to the optimization problem depends on the discount rate.

Proposition 2 *Suppose that Assumption 1 holds and the optimal job design is task separation. Then the optimal relational contract satisfies the following property.*

1. *For $r < (a_1^2 m_2 + a_2^2 m_1)/2(a_1 m_2 + a_2 m_1)$, the dynamic enforcement constraint is not binding.*
2. *For $r \in [(a_1^2 m_2 + a_2^2 m_1)/2(a_1 m_2 + a_2 m_1), A/2)$, the dynamic enforcement constraint is binding and $\beta_i > 0$ for both $i = 1, 2$.*
3. *For $r \in [A/2, a_1)$, the dynamic enforcement constraint is binding, $\beta_1 > 0$, and $\beta_2 = 0$.*

[13]Similar to task bundling, we ignore possibilities of negative bonuses, which are never strictly optimal.

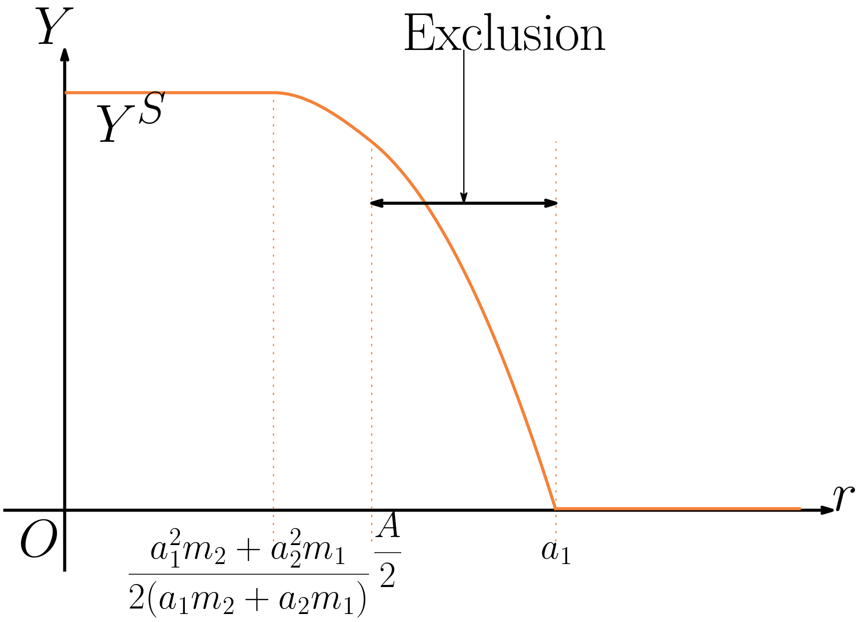

Figure 3: Principal's Payoff under Task Separation

4. *For* $r \geq a_1$, $\beta_i = 0$ *for both* $i = 1, 2$.

5. *The principal's payoff* Y^S *is*

   (a) *non-increasing in* $r > 0$*; and*

   (b) *strictly decreasing in* $r \in ((a_1^2 m_2 + a_2^2 m_1)/2(a_1 m_2 + a_2 m_1), a_1)$.

Figure 3 illustrates the principal's payoff on the optimal relational contract under task separation, denoted by Y^S. Similar to task bundling, the principal's payoff is monotonic in the parties patience. One specific feature is that, for intermediate discount rate such that $A/2 \leq r < a_1$, the optimal contract exhibits *task exclusion*, under which an agent does not exert any effort on any assigned tasks. Under Assumption 1, if tasks are excluded, then those assigned to agent 2 must be excluded. In this sense, an agent with higher a_i is more important than the other agent with lower a_i.

4 Optimal Job Design

In this section, we derive the optimal job design by comparing the optimal principal's payoff of each job design. Before providing the formal result, we first explain the key trade-off behind the result and the interpretation of the parameters introduced above.

4.1 The Trade-Off of Job Design

As the existing literature has already pointed out, separating the tasks mitigates misallocation of effort caused by multitasking problems with distorted performance measurements. From the incentive compatibility constraint under task bundling (3), the pair of effort chosen by agent 1 must be $\boldsymbol{e} = \overline{\beta}\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}$, namely, a point on vector $\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1} \equiv (\mu_1/\gamma_1, \ldots, \mu_N/\gamma_N)$. On the other hand, task separation may implement other pairs of effort. For explanatory simplicity, suppose $\mathcal{N}_1 = \{1, 2, \ldots, N_1\}$ and $\mathcal{N}_2 = \{N_1 + 1, \ldots, N\}$ for some $N_1 = 1, \ldots, N-1$. Then the incentive compatibility constraint under task separation (7) implies that agents 1 and 2 choose a pair of effort $\boldsymbol{e}_1 = \beta_1(\mu_1/\gamma_1, \ldots, \mu_{N_1}/\gamma_{N_1})$ and $\boldsymbol{e}_2 = \beta_2(\mu_{N_1+1}/\gamma_{N_1+1}, \ldots, \mu_N/\gamma_N)$. The effort choice of each agent is dependent on his own bonus but independent of the other's bonus. In other words, by choosing a pair of bonuses (β_1, β_2) appropriately, the principal can implement a pair of effort $\boldsymbol{e}$ that is on the plane with basis $\boldsymbol{\mu}_1\boldsymbol{\Gamma}^{-1} \equiv (\mu_1/\gamma_1, \ldots, \mu_{N_1}/\gamma_{N_1}, 0, \ldots, 0)$ and $\boldsymbol{\mu}_2\boldsymbol{\Gamma}^{-1} \equiv (0, \ldots, 0, \mu_{N_1+1}/\gamma_{N_1+1}, \ldots, \mu_N/\gamma_N)$. Note that vector $(\mu_1/\gamma_1, \ldots, \mu_N/\gamma_N)$ must be on this plane, meaning that all the pairs of effort that can be implemented under task bundling can also be implemented under task separation as long as a pair of bonuses (β_1, β_2) can be appropriately chosen. Since the principal's payoff $Y(\boldsymbol{e})$ is maximized at $\boldsymbol{e} = \boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1} \equiv (\alpha_1/\gamma_1, \ldots, \alpha_N/\gamma_N)$, which is in general not on vector $\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}$, task separation may help the principal implement a more desirable effort pair.

Nevertheless, task separation contains a drawback of serious commitment problems. As the dynamic enforcement constraints (2) and (6) exhibit, the amount of discretionary bonuses must be limited due to the principal's reneging temptation. Then the principal must implement a desirable effort pair with smaller bonuses. Now suppose that the principal can commit to β units of the bonus to pay. If this bonus is paid to agent 1 under task separation, then the incentive compatibility constraint under task separation (7) implies that agent 1 chooses a pair of effort $\beta_1(\mu_1/\gamma_1, \ldots, \mu_{N_1}/\gamma_{N_1})$ while agent 2 chooses zero effort on all of his assigned tasks. On the other hand, under task bundling, the incentive compatibility constraint under task bundling (3) implies that agent 1 chooses a pair of effort $\beta_1\boldsymbol{\mu}\boldsymbol{\Gamma} \equiv \beta_1(\mu_1/\gamma_1, \ldots, \mu_{N_1}/\gamma_{N_1}, \mu_{N_1+1}/\gamma_{N_1+1}, \ldots, \mu_N/\gamma_N)$, meaning that the agent performs the tasks as if a pair of bonus (β_1, β_1) is promised under task separation. In other words, if a pair of effort can be implemented under task bundling, then implementing the same effort pair requires a greater amount of the bonus under task separation.

4.2 Interpretation of Parameters

We now provide interpretation of parameters A, M, a_i, and m_i, which affect the optimal relational contract under each job design. First, consider task bundling. Taking the first derivative of (3) with respect to $\overline{\beta}$ yields $de_n/d\overline{\beta} = \mu_n/\gamma_n$ for each $n \in \mathcal{N}$. Since (3) is the incentive compatibility constraint described by the first order condition, the term μ_n/γ_n expresses the incremental increase of effort on task n by an incremental increase of the bonus. Furthermore, the principal's marginal benefit of effort on task n is $\partial f(\boldsymbol{e})/\partial e_n = \alpha_n$. Since under task bundling one unit of bonus increases effort by μ_n/γ_n for each $n \in \mathcal{N}$, parameter $A \equiv \boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}' = \sum_{n\in\mathcal{N}} \alpha_n\mu_n/\gamma_n$ is interpreted as the principal's *marginal benefit of the bonus*. Similarly, since the marginal success probability of effort on task n is $\partial p(\boldsymbol{e})/\partial e_n = \mu_n$, parameter $M \equiv \boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}' = \sum_{n\in\mathcal{N}} \mu_n^2/\gamma_n$ is interpreted as the *marginal success probability of the bonus* under task bundling.

It is possible to obtain a similar interpretation of a_i and m_i. The first derivative of (7) with respect to β_i is $de_n/d\beta_i = \mu_n/\gamma_n$, meaning again the marginal effort of the bonus. Nevertheless, since one unit of the bonus to agent i increases effort only for the assigned tasks $n \in \mathcal{N}_i$ under task separation, an incremental increase of the bonus to agent i increases the principal benefit only by $a_i \equiv \boldsymbol{\alpha}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}_i' = \sum_{n\in\mathcal{N}_i} \alpha_n\mu_n/\gamma_n$ and the success probability only by $m_i \equiv \boldsymbol{\mu}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}_i' = \sum_{n\in\mathcal{N}_i} \mu_n(\mu_n/\gamma_n)$. In other words, parameters a_i and m_i are interpreted as the marginal benefit and success probability through agent i under task separation.

The objective functions in the optimization problems (4) and (8) indicate that the optimal discretionary bonus should take into account the marginal benefits of the bonus, A and a_i, and the marginal success probabilities of the bonus, M and m_i. Intuitively, the former measures the degree of the principal's benefit from an incremental increase of the bonus while the latter measures the agent's effort sensitivity to the bonus. The optimum points of each objective function, A/M and $(a_1/m_1, a_2/m_2)$, respectively, imply that the bonus should be higher if the marginal benefit is high and the marginal success probability is low. It is intuitive to understand that the bonus should be higher when the marginal benefit of bonus, A and a_i is high. On the other hand, if M or m_i is high and the principal pays a large bonus, then in order to induce the successful signal, the agent exerts excessive effort relative to the surplus maximizing level. Therefore, if the agent is sensitive to the bonus, then the principal should decrease the bonus.

Under task separation, each agent has his own marginal benefit a_i and marginal success probability m_i according to the assigned tasks so that the desirable bonuses,

a_1/m_1 and a_2/m_2, are in general different. Based on these parameters, the principal may pay discretionary bonuses to each agent in a discriminative way. In contrast to task separation, such bonus discrimination is infeasible under task bundling since all the tasks are assigned to a single agent and the evaluation is based only on a single aggregated performance measurement. Hence, as already pointed out in Section 4.1, the allocation of effort is distorted more under task bundling.

The degree of misallocation of effort is determined by a_1/a_2, the ratio of the agents' marginal benefits, and m_1/m_2, the ratio of the agents' marginal success probabilities. As an extreme case, when a_1/a_2 is equal to m_1/m_2, we immediately obtain $a_1/m_1 = a_2/m_2$, which means that the bonuses do not have to be discriminated even under task separation. However, as a_1/a_2 departs from m_1/m_2, since $a_1/m_1 \neq a_2/m_2$, the principal prefers to pay the discriminative bonuses to the agents under task separation. This implies that misallocation of effort under task bundling is serious relative to task separation. Based on these observations, we classify circumstances into three cases depending on the distance between the ratios a_1/a_2 and m_1/m_2.

Definition 1 *The tasks are:*

1. *(perfectly) balanced when* $a_1/a_2 = m_1/m_2$*;*
2. *weakly unbalanced when* $a_1/a_2 \neq m_1/m_2$ *and* $1 \leq a_1/a_2 \leq 2m_1/m_2 + 1$*; and*
3. *strongly unbalanced when* $a_1/a_2 > 2m_1/m_2 + 1$.

Note that the interpretation here may be useful for other multitasking models with a general number of tasks. Many of theoretical articles on multitasking problems and job design usually assume a specific number of tasks, actually two tasks in many cases.[14] Actually, when $\#\mathcal{N} = 2$, we obtain a simple intuition that the tasks are perfectly balanced if and only if $\alpha_1/\alpha_2 = \mu_1/\mu_2$, or equivalently $\boldsymbol{\alpha}$ and $\boldsymbol{\mu}$ are linearly dependent, meaning that the direction of the marginal success probability of effort is perfectly alight to that of the principal's marginal benefit of effort. When there are three or more tasks, unfortunately it is impossible to obtain such an intuitive condition on the degree of the multitasking problem simply based on the direction of $\boldsymbol{\alpha}$ and $\boldsymbol{\mu}$. Nevertheless, our procedure suggests that incentive problems with a general number of multiple tasks may be successfully analyzed by summarizing numerous parameters into the

[14] Itoh (1994, 2001) and Kragl and Schöttner (2014) assume two tasks in total. Schöttner (2008) assumes three tasks. In articles being interested in issues of team, the number of tasks are typically assumed to be four (Besanko et al., 2005; Corts, 2007; Mukherjee and Vasconcelos, 2011).

marginal benefit of the bonus and success probability of the bonus for each agent. Furthermore, how the multitasking incentive problem is serious may be transformed into the degree of bonus discrimination.

As seen in the next section, the case of balanced tasks are exceptional in that task bundling weakly dominates task separation. In other cases, there is a trade-off between task bundling and task separation and the former is preferred if and only if the parties are impatient.

4.3 Balanced Tasks

When the tasks are perfectly balanced, task bundling weakly dominates task separation.

Proposition 3 *Suppose that Assumption 1 holds and the tasks are perfectly balanced.*

1. *For* $r \in (A/4, A)$, $Y^B > Y^S$.
2. *Otherwise,* $Y^B = Y^S$.

Figure 4 illustrates the principal's optimal payoffs under task bundling, Y^B, and under task separation, Y^S, in a unified diagram. When the tasks are balanced, effort allocation cannot be strictly improved at all by splitting the tasks: the optimal effort allocation implemented under task separation can also be implemented under task bundling. In addition, the required bonus for implementing the targeted effort can be smaller under task bundling than task separation. As a result, task separation never outperforms task bundling.

4.4 Unbalanced Tasks

When the tasks are (weakly or strongly) unbalanced, task separation can mitigate misallocation of effort caused under task bundling. As a result, task separation is actually optimal if the parties are sufficiently patient.

Proposition 4 *Suppose that Assumption 1 holds and the tasks are weakly unbalanced.*

1. *For* $r < \max_{i=1,2}\{m_i/a_i\}A^2/4M$, $Y^S > Y^B$.
2. *For* $r \in (\max_{i=1,2}\{m_i/a_i\}A^2/4M, A)$, $Y^S < Y^B$.
3. *Otherwise,* $Y^S = Y^B$.

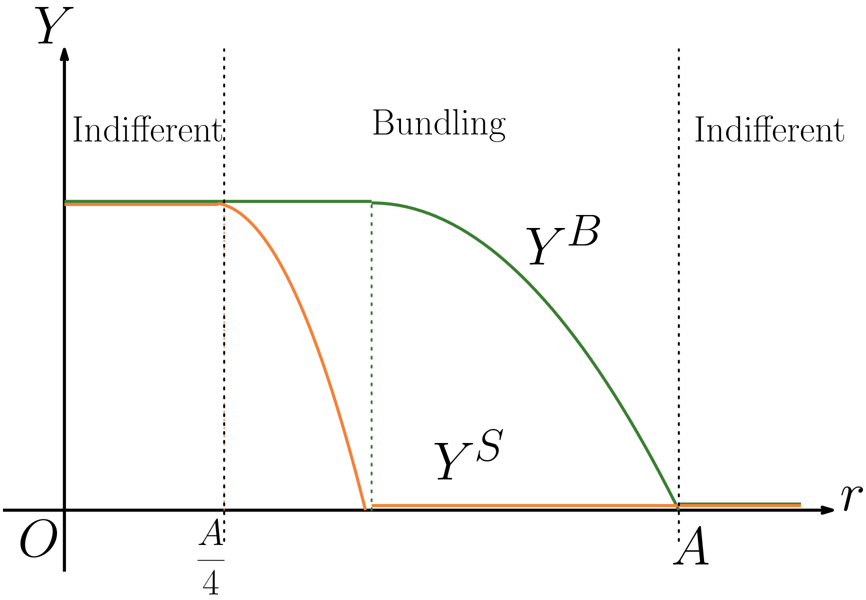

Figure 4: Principal's Payoff of Balanced Tasks

Proposition 5 *Suppose that Assumption 1 holds and the tasks are strongly unbalanced.*

1. *For* $r < a_1 - a_2 m_1/m_2$*,* $Y^S > Y^B$*. Furthermore, for* $A/2 \leq r < a_1 - a_2 m_1/m_2$*, agent* 2 *exerts no effort.*

2. *For* $r \in (a_1 - a_2 m_1/m_2, A)$*,* $Y^S < Y^B$*.*

3. *Otherwise,* $Y^S = Y^B$*.*

Figures 5 and 6 illustrate the principal's optimal payoffs in cases of weakly and strongly unbalanced tasks, respectively. In both cases, task separation achieves a higher payoff for the principal than task bundling as long as appropriate bonuses are feasible. When the parties are sufficiently patient, the principal's commitment to the informal bonus is not a concern and then task separation with ideal bonus payments is optimal. However, as the parties become impatient, such a bonus payment becomes incredible. Then, instead of task separation, task bundling is the optimal job design due to its advantage of self-enforcement.

In cases of strongly unbalanced tasks, task exclusion emerges as the optimal job design for intermediate discount rate. As Figure 6 illustrates, $a_1 - a_2 m_1/m_2$, the threshold value of the discount rate between task separation and task bundling is greater than $A/2$. Since Proposition 2 shows that the optimal contract under task separation exhibits task exclusion for $r \in [A/2, a_1)$, the principal chooses task exclusion for $r \in [A/2, a_1 - a_2 m_1/m_2)$. This means that the principal optimally chooses to exclude the tasks though

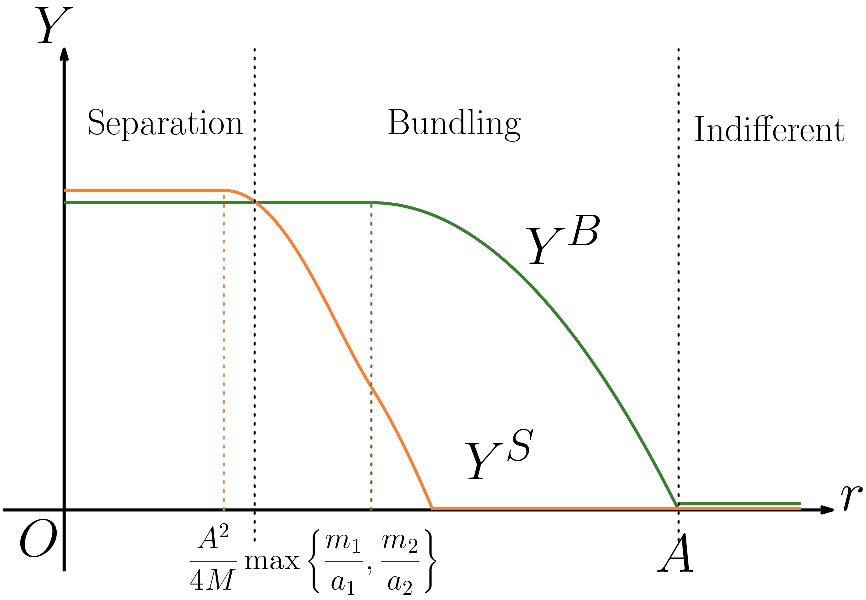

Figure 5: Principal's Payoff of Weakly Unbalanced Tasks

she can induce positive effort on the excluded tasks through task bundling. As pointed out in Section 4.1, assigning additional tasks by choosing task bundling induces the agent to exert positive effort on the additionally assigned tasks in $\mathcal{N}_2$ without any extra bonuses. However, the effort level on the excluded tasks is inefficiently high under task bundling. Formally, this is confirmed from the following lemma.

Lemma 3 *Suppose that Assumption 1 holds, the tasks are strongly unbalanced, and* $r \in [A/2, a_1 - a_2m_1/m_2)$*. Let* $\overline{\beta}$ *be the optimal solution to (4) and* (β_1, β_2) *be the optimal solution to (8). Then* $\overline{\beta} < \beta_1$ *and* $a_2\overline{\beta} - m_2\overline{\beta}^2/2 < 0$*.*

Lemma 3 implies that when the tasks are strongly unbalanced and $r \in [A/2, a_1 - a_2m_1/m_2)$, under task bundling, the contribution to the principal's benefit from tasks in $\mathcal{N}_2$, i.e. $a_2\overline{\beta} - m_2\overline{\beta}^2/2$, is negative. The intuition is as follows. Since a_1 and m_2 are quite high relative to a_2 and m_1, the principal is almost exclusively interested in inducing effort on tasks in $\mathcal{N}_1$. Under task bundling, the bonus $\overline{\beta}$ is lower than β_1, which is the bonus paid to agent 1 under task separation, since the principal attempts to reduce effort on tasks in $\mathcal{N}_2$. However, the agent still chooses excess effort on tasks in $\mathcal{N}_2$ since such tasks are effective to increase the success probability of the performance measurement. Therefore it is optimal for the principal to deliberately exclude tasks in $\mathcal{N}_2$, by which the excess effort is prevented and a high powered incentive can be provided for agent 1. Nevertheless, the principal chooses task bundling to induce positive effort on tasks in $\mathcal{N}_2$ if the parties become further less patient so that $r > a_1 - a_2m_1/m_2$. This is because

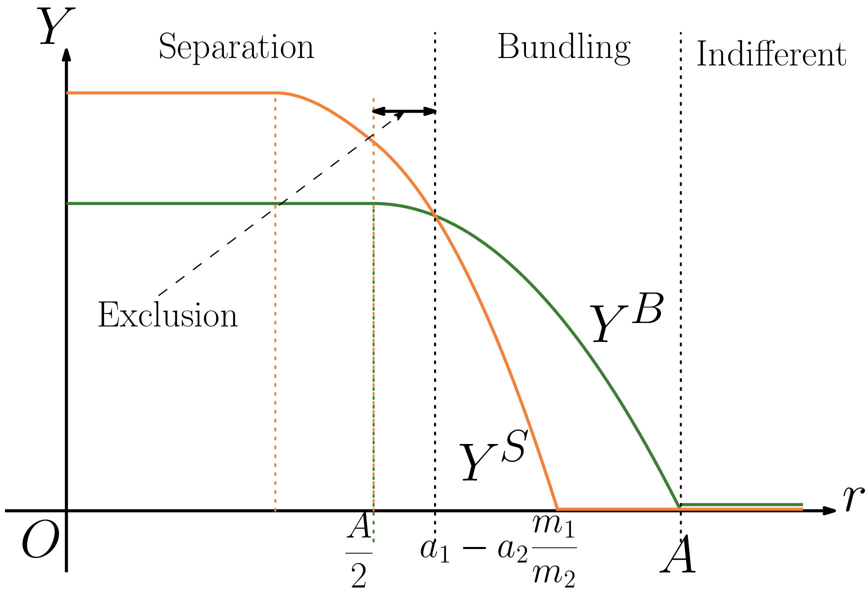

Figure 6: Principal's Payoff of Strongly Unbalanced Tasks

the bonus $\overline{\beta}$ is so small that the effort level on tasks $n \in \mathcal{N}_2$ is no longer excess relative to the principal's desired level.

The optimality of task exclusion is also pointed out by Kragl and Schöttner (2014), who consider a one-shot relationship in which the performance measurement is verifiable and the agents are protected by wage floors. They find a monotonic tendency in that a task is inclined to be excluded as the wage floor rises up given the degree of distortion of the performance measurement fixed. Increase of wage floors is a source of inefficiency of incentive provision due to increase of positive rents received by the agent. Given that the performance measurement is sufficiently distorted, hiring only a single agent and excluding a task can reduce the rents and mitigate distortion of the effort allocation. In our setup of relational contracting, by contrast, the relationship between patience and task exclusion is not monotonic. As seen above, being more impatient does not necessarily imply task exclusion and the parties who are sufficiently impatient choose task bundling even though the tasks are strongly unbalanced.[15]

4.5 Effect of the Balancedness

We have seen the optimal job design for each r, patience of the parties. The optimal job design can also be characterized by the balancedness of the tasks. To derive the result, it is helpful to introduce new parameters: for each $i = 1, 2$, let $\rho_{Ai} \equiv a_i/A$, $\rho_{Mi} \equiv m_i/M$.

[15] Nevertheless, there is a monotonic relationship between the parties' patience and the amount of the bonus received by agent 2.

Since $\sum_{i=1}^{2} a_i = A$, $\sum_{i=1}^{2} m_i = M$, and all of these parameters are positive, Assumption 1 implies $1 > \rho_{A1} \geq 1/2 \geq \rho_{A2} > 0$. A nice property of expression of ρ_{Ai} and ρ_{Mi} is that the balancedness is comprehensively described only by ρ_{A1} and ρ_{M1}.[16]

Lemma 4 *Under Assumption 1, the tasks are:*

1. *(perfectly) balanced when* $\rho_{A1} = \rho_{M1}$*;*
2. *weakly unbalanced when* $\rho_{A1} \neq \rho_{M1}$ *and* $1/2 \leq \rho_{A1} \leq (1 + \rho_{M1})/2$*; and*
3. *strongly unbalanced when* $\rho_{A1} > (1 + \rho_{M1})/2$*.*

The following proposition provides another illustration of the condition for the optimal job design.

Proposition 6 *Suppose that Assumption 1 holds.*

1. *When* $\rho_{M1} < 1/2$*:*
   (a) *for* $r \leq (1 - \rho_{M1})A/2$*,* $Y^S > Y^B$ *for any* $\rho_{A1} \in [1/2, 1)$*; and*
   (b) *for* $r \in ((1 - \rho_{M1})A/2, A)$*, there exists a threshold* $\hat{\rho}_{A1} \in (1/2, 1)$ *such that* $Y^S \gtreqless Y^B$ *if and only if if and only if* $\rho_{A1} \gtreqless \hat{\rho}_{A1}$*.*
2. *When* $\rho_{M1} \geq 1/2$*:*
   (a) *for* $r \leq A/4$*,* $Y^S > Y^B$ *for any* $\rho_{A1} \in [1/2, 1) \setminus \{\rho_{M1}\}$*;*
   (b) *for* $r \in (A/4, \rho_{M1}A/2]$*, there exist two thresholds* $\underline{\rho}_{A1} \in [1/2, \rho_{M1})$ *and* $\overline{\rho}_{A1} \in (\rho_{M1}, 1]$ *such that* $Y^S > Y^B$ *(resp.* $Y^S < Y^B$*) if and only if* $\rho_{A1} < \underline{\rho}_{A1}$ *or* $\rho_{A1} > \overline{\rho}_{A1}$ *(resp.* $\rho_{A1} \in (\underline{\rho}_{A1}, \overline{\rho}_{A1})$*); and*
   (c) *for* $r \in (\rho_{M1}A/2, A)$*, there exists a threshold* $\hat{\rho}_{A1} \in (\rho_{M1}, 1)$ *such that* $Y^S \gtreqless Y^B$ *if and only if* $\rho_{A1} \gtreqless \hat{\rho}_{A1}$*.*
3. *For* $r \in [A/2, A)$*, if* $\rho_{A1} > \hat{\rho}_{A1}$*, where* $\hat{\rho}_{A1}$ *is defined above, then tasks in* $\mathcal{N}_2$ *are excluded.*

Proposition 6 is explained in an intuitive way by Figures 7 and 8, which illustrate the optimal job design characterized by r and ρ_{A1} given ρ_{M1} fixed. In both figures, the dash-dot curves, namely,

$$r = \hat{r}(\rho_{A1}) \equiv \begin{cases} \max\left\{\dfrac{\rho_{M1}}{\rho_{A1}}, \dfrac{1-\rho_{M1}}{1-\rho_{A1}}\right\}\dfrac{A}{4} & \text{if } \rho_{A1} \in \left[\dfrac{1}{2}, \dfrac{1+\rho_{M1}}{2}\right], \\ \dfrac{A(\rho_{A1}-\rho_{M1})}{1-\rho_{M1}} & \text{if } \rho_{A1} \in \left(\dfrac{1+\rho_{M1}}{2}, 1\right), \end{cases} \tag{9}$$

[16] The proof of Lemma 4 is easy and omitted.

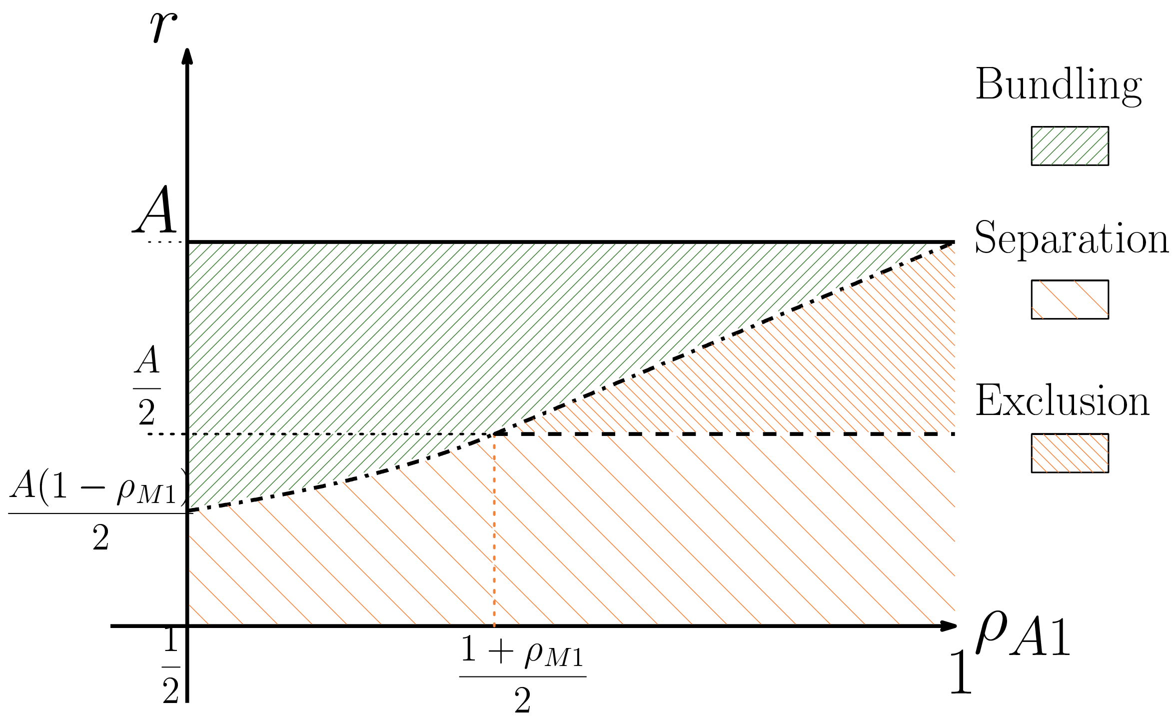

Figure 7: Optimal Job Design when $\rho_{M1} < 1/2$

are the boundary of the optimal job design. Consistent to the results shown in the previous subsections, task separation is preferred for lower area in the figures. Here we additionally see that task bundling is inclined to be preferred as the tasks are getting less unbalanced. Specifically, when ρ_{A1} approaches ρ_{M1}, the range of r in which task bundling is optimal becomes broader. Note that Lemma 4 provides an interpretation that the tasks becomes more unbalanced as ρ_{A1} departs from ρ_{M1}. As pointed out in Section 4.1, when the tasks becomes more unbalanced, discrimination of bonuses between the agents under task separation has a large room for improvement of efficiency. Thus, the principal facing with unbalanced tasks is inclined to choose task separation.

5 Interaction with Explicit Incentives

5.1 Extended Model

The baseline model so far assumes that there is no verifiable performance measurement and only an implicit incentive is provided. This simplification allows us to captures the key trade-off of job design in an intuitive way. Nevertheless, in reality, implicit incentive provision is often combined with explicit incentive contracts based on verifiable

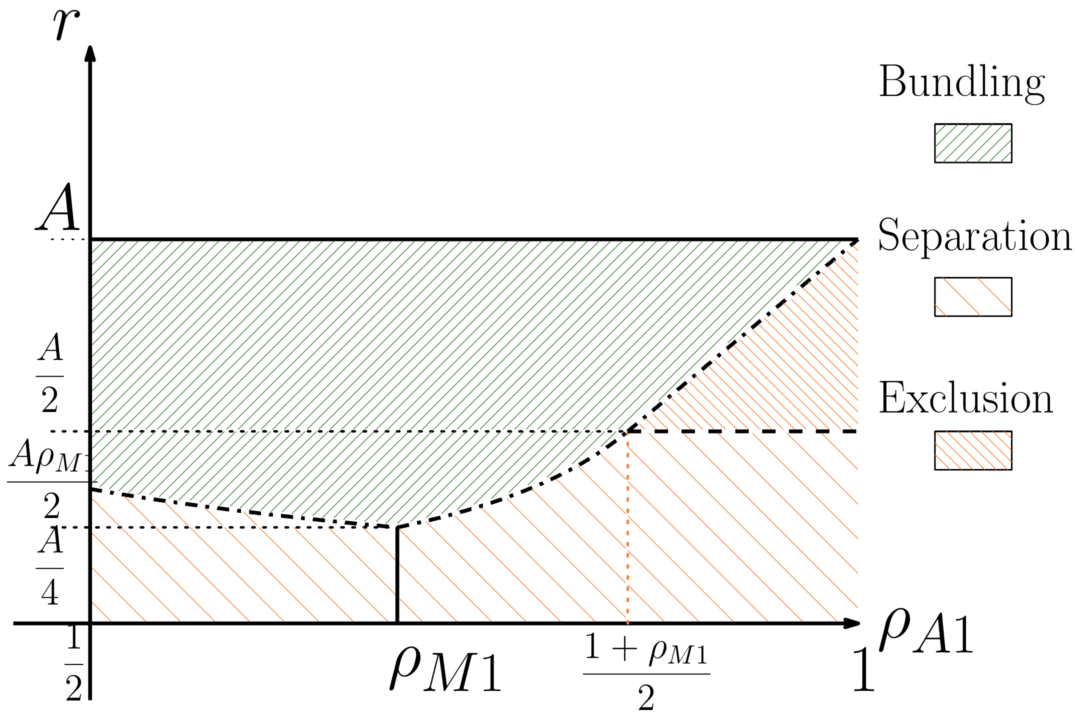

Figure 8: Optimal Job Design when $\rho_{M1} \geq 1/2$

performance measurements. For instance, in cases of clinical development mentioned in Section 1, contractual parties are often rewarded based on contractible performances such as the time of the delivery of the data.

This section discusses how the presence of explicit incentive contracts affects relational contracts and job design. We extend our baseline model such that, in addition to nonverifiable performance measurement x_t, there further exists a verifiable performance measurement z_t stochastically generated after effort choice in each period t. The verifiable performance measurement is either $z_t = S$ with probability $h(\boldsymbol{e}_t) \equiv \min\{\sum_{n \in \mathcal{N}} \eta_n e_{nt}, 1\}$; or $z_t = F$ with probability $1 - h(\boldsymbol{e}_t)$, where $\eta_n > 0$ for all $n \in \mathcal{N}$. Similar to α_n and μ_n, η_ns are assumed to be sufficiently small relative to γ_ns in order for the first order approach to be valid. Let $\boldsymbol{\eta} \equiv (\mu_1, \ldots, \mu_N)$ and $\boldsymbol{\eta}_i \equiv (\nu_{1i}\mu_1, \ldots, \nu_{Ni}\mu_N)$ for $i = 1, 2$.

The principal offers a formal contract that can now be contingent on the verifiable performance measurement z_t. Then a formal contract specifies a transfer rule $(w_{it}(S), w_{it}(F)) \in \mathbb{R}^2$. In addition, as before, the principal informally promises a discretionary bonus $(b_{it}(S), b_{it}(F))$ that is contingent on the nonverifiable performance measurement x_t.

Throughout the section, we focus on the following parametric assumption on $\boldsymbol{\eta}$.

Assumption 2 1. *The following inequalities are satisfied:*

$$\frac{\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'}{\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'} \geq \frac{\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'}{\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'}; \tag{10}$$

and for $i = 1, 2$,

$$\frac{\boldsymbol{\alpha}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}_i'}{\boldsymbol{\eta}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}_i'} \geq \frac{\boldsymbol{\alpha}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'}{\boldsymbol{\eta}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'}. \tag{11}$$

2. *$\boldsymbol{\alpha}$ and $\boldsymbol{\eta}$ are linearly independent.*

Roughly speaking, inequalities (10) and (11) imply that explicit incentives are not so strongly substitutes for implicit incentives that the agents never receive negative informal bonuses when the nonverifiable signal reveals success ($x_t = S$). The second condition implies that providing informal bonuses is a substantial issue. Actually, if $\boldsymbol{\alpha}$ and $\boldsymbol{\eta}$ are linearly dependent, then the principal can implement the first best effort level only through explicit contracts under both task bundling and separation.

In the following, we highlight the optimal job design in the extended model. The detail of the formal analysis, including further interpretations of Assumption 2, is found in Appendix B.

5.2 Optimal Job Design

5.2.1 Spot Contracting

Before proceeding to the characterization of optimal relational contracts, as a benchmark, we first consider spot contracting, by which we mean an equilibrium outcome when the parties interact only once. When the relationship is static, the parties cannot utilize any informal bonuses for incentive provision. Nevertheless, the principal can yield a positive payoff through explicit incentives.

The payoff of spot contracting differs between task bundling and task separation since formal incentive contracts affect the multitasking incentive even without informal bonuses. Actually, the principal's payoffs of spot contracting under task bundling and task separation, denoted by $\underline{y}^B$ and $\underline{y}^S$, respectively, are characterized as

$$\underline{y}^B \equiv \frac{(\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}')^2}{2\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'}, \qquad \underline{y}^S \equiv \sum_{i=1}^{2} \frac{(\boldsymbol{\alpha}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i')^2}{2\boldsymbol{\eta}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'}.$$

By comparing these values, it is easy to show that task separation always achieves an weakly higher payoff than task bundling.

Lemma 5 $\underline{y}^S \geq \underline{y}^B$*, where the inequality holds with strict inequality if and only if*

$$\frac{\alpha_1 \boldsymbol{\Gamma}^{-1} \eta_1'}{\eta_1 \boldsymbol{\Gamma}^{-1} \eta_1'} \neq \frac{\alpha_2 \boldsymbol{\Gamma}^{-1} \eta_2'}{\eta_2 \boldsymbol{\Gamma}^{-1} \eta_2'}. \tag{12}$$

The argument on the optimal job design of spot contracting is consistent to Kragl and Schöttner (2014). Since there is no limited liability constraint in our model, as they argue, separating tasks into multiple agents yields no additional costs. Here we confirm that their argument is true even if the number of the tasks are three or more.

5.2.2 Optimality of Task Separation

Now we investigate optimal relational contracts in the extended model. As in the baseline model, under each job design, an optimal equilibrium is without loss of generality stationary and the optimal relational contract can be characterized by solving the optimization problem. Hereafter let $\hat{Y}^B$ and $\hat{Y}^S$ be the principal's optimal payoffs under task bundling and task separation, respectively, when a verifiable performance measurement is available. By comparing $\hat{Y}^B$ and $\hat{Y}^S$, we show that when explicit incentives are available, task separation is at least weakly optimal when the parties are sufficiently impatient as well as sufficiently patient.

Proposition 7 *For sufficiently small r or sufficiently large r, $\hat{Y}^S \geq \hat{Y}^B$.*

When the parties are sufficiently impatient, implicit incentives are not feasible at all and explicit incentive contracts solely determine the performance. Then, comparison of the payoffs of spot contracting immediately implies that task separation yields a higher payoff.

As in the baseline model, the closed form expression of the optimal payoff can be derived under each job design. Then in principle, the optimal job design can be characterized by directly comparing the optimal payoff under each job design. Nevertheless, unless there is additional restriction on parameters, it is difficult to state a general qualitative result on the optimal job design. Hereafter, we make additional restrictions on the parameters to illustrate the pattern of the optimal job design in an intuitive way.

5.2.3 Quasi-Symmetric Environment

We first make the following 'quasi-symmetric' assumption.

Assumption 3

$$\frac{\mu_1 \Gamma^{-1} \eta_1'}{\eta_1 \Gamma^{-1} \eta_1'} = \frac{\mu_2 \Gamma^{-1} \eta_2'}{\eta_2 \Gamma^{-1} \eta_2'}.$$

The quasi-symmetry assumption is satisfied when, for instance, the assigned tasks satisfy the symmetric property in the following sense: N is an even number, $\mathcal{N}_1 = \{1, \ldots, N/2\}$, $(\mu_1, \ldots, \mu_{N/2}) = (\mu_{N/2+1}, \ldots, \mu_N)$, $(\eta_1, \ldots, \eta_{N/2}) = (\eta_{N/2+1}, \ldots, \eta_N)$, and $(\gamma_1, \ldots, \gamma_{N/2}) = (\gamma_{N/2+1}, \ldots, \gamma_N)$. Given this symmetric assumption, under task separation the agents implement the same effort allocation if both explicit and implicit incentive schemes have the identical structure between the agents.

With an additional assumption that the value of the spot contracting under task separation is strictly greater than under task bundling, in quasi-symmetric environments, the optimal job design is comprehensively characterized as follows.

Proposition 8 *Suppose that Assumptions 2 and 3 and (12) hold.*

1. *If $\boldsymbol{\mu}$ and $\boldsymbol{\eta}$ are linearly dependent, then $\hat{Y}^S > \hat{Y}^B$ for all r.*
2. *If $\boldsymbol{\mu}$ and $\boldsymbol{\eta}$ are linearly independent, then there exists a value, denoted by $\hat{y}$, such that the optimal relational contract satisfies the following.*
   - *(a) When $\underline{y}^S - \underline{y}^B \leq \hat{y}$, there exist $\underline{r} > 0$ and $\bar{r} \geq \underline{r}$ such that:*
     - *i. $\hat{Y}^B > \hat{Y}^S$ for $r \in (\underline{r}, \bar{r})$;*
     - *ii. $\hat{Y}^B = \hat{Y}^S$ for $r = \underline{r}, \bar{r}$; and*
     - *iii. $\hat{Y}^B < \hat{Y}^S$ otherwise.*
   - *(b) When $\underline{y}^S - \underline{y}^B > \hat{y}$, $\hat{Y}^B < \hat{Y}^S$ for all r.*

Figures 9 and 10 illustrate the principal's payoff characterized by Proposition 8. As in Proposition 7, task separation is optimal if the parties are sufficiently patient or sufficiently myopic. In addition, Proposition 8 states that there may or may not be an intermediate range of r such that task bundling is strictly preferred. Recall that separating tasks can mitigate misallocation of effort through explicit incentives as well as implicit incentives. When the difference of the values of spot contracting is sufficiently large or $\boldsymbol{\mu}$ and $\boldsymbol{\eta}$ are linearly dependent, separating the tasks has a substantial impact

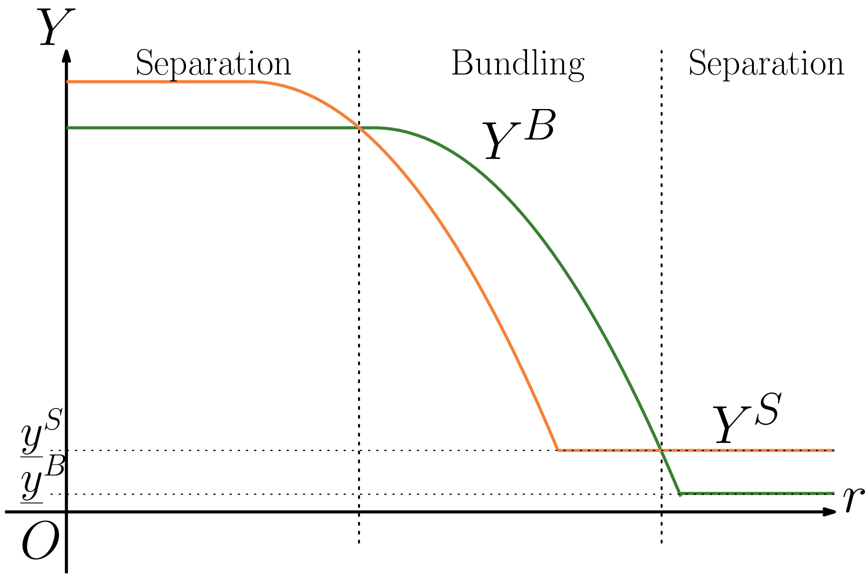

Figure 9: Principal's Payoff when $\underline{y}^S - \underline{y}^B \leq \hat{y}$

on mitigating misallocation of effort only through explicit incentives and does not need a large amount of informal bonuses to improve efficiency further. Then the benefit of saving informal bonus payments from bundling the tasks is negligible and, as a result, task bundling cannot be strictly preferred for any r.

5.2.4 When Patient Parties Prefer Task Bundling?

We now make a different assumption, by which we clarify the relationship to the similar model studied by Schöttner (2008). As we pointed out in Section 1, in her model, task bundling is preferred to task separation if and only if the parties are patient, which is the opposite implication to our main result. The key assumption behind her result is that the principal's benefit is nonverifiable but observable to the parties. In other words, the principal's benefit in our model coincides with nonverifiable but observable performance measurement x_t.

When the principal's benefit is observable to the parties, the principal can arrange informal bonuses contingent on the principal's benefit. Given such informal bonuses and effort level $(e_1, \ldots, e_N)$, the agents believe that the probability of receiving the informal bonus, expressed by $\sum_{n \in \mathcal{N}} \mu_n e_n$, is equal to the probability of yielding benefit 1, expressed by $\sum_{n \in \mathcal{N}} \alpha_n e_n$. In our model, the agents have such a belief if vector $\boldsymbol{\alpha}$ is equal to vector $\boldsymbol{\mu}$. Here, we make a weaker assumption as follows.

Assumption 4 *$\boldsymbol{\alpha}$ and $\boldsymbol{\mu}$ are linearly dependent.*

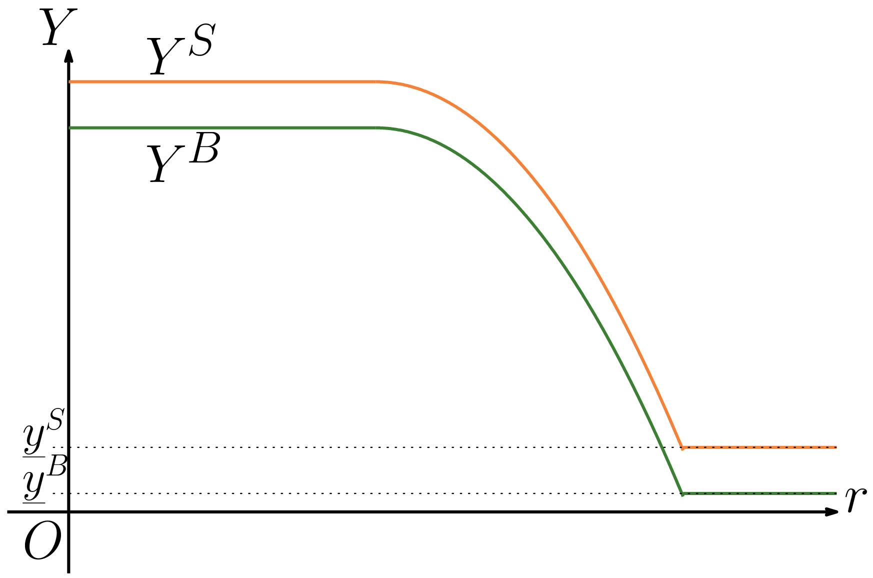

Figure 10: Principal's Payoff when $\underline{y}^S - \underline{y}^B > \hat{y}$

In addition to the assumption that $\boldsymbol{\alpha} = \boldsymbol{\mu}$, Schöttner (2008) focuses on the case of three tasks: i.e. $\#\mathcal{N} = 3$. Nevertheless, her result on the optimal job design holds without restrictions on the number of the tasks.

Proposition 9 *Suppose that Assumption 4 holds and $\boldsymbol{\alpha}$ and $\boldsymbol{\eta}$ are linearly independent.*

1. *If (12) is violated and $\boldsymbol{\alpha}_i$ and $\boldsymbol{\eta}_i$ are linearly dependent for some $i = 1, 2$, then $\hat{Y}^B = \hat{Y}^S$ for all r.*
2. *Otherwise, there exist $\underline{r} > 0$ and $\overline{r} > \underline{r}$ such that:*
   (a) for $r \leq \underline{r}$ or $r = \overline{r}$, $\hat{Y}^B = \hat{Y}^S$;
   (b) for $r \in (\underline{r}, \overline{r})$, $\hat{Y}^B > \hat{Y}^S$; and
   (c) for $r > \overline{r}$, $\hat{Y}^B \leq \hat{Y}^S$, where strict inequality holds if and only if (12) is satisfied.

Figure 11 illustrates the main argument in Proposition 9 as well as the result on the optimal job design in Schöttner (2008). The weaker assumption yields the same result as Schöttner (2008) due to the following reason. The assumption in Schöttner (2008) that the principal's benefit is nonverifiable but observable to the parties implies that the principal's observable benefit is a sufficiently informative performance measurement in that, regardless of job design, the principal can induce the first best effort pair from the agents as long as the principal provides a right incentive based on the nonverifiable performance measurement. Even if the nonverifiable performance measurement is not perfectly correlated to the principal's benefit, the agents' incentive structure is still the

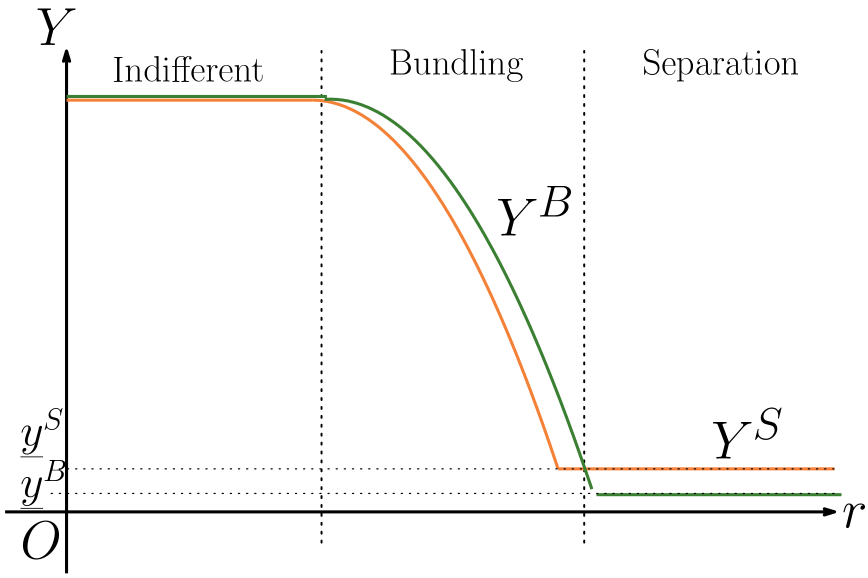

Figure 11: Principal's Payoff under Assumption 4

same as long as $\boldsymbol{\mu}$, the effect of effort on the success of the nonverifiable performance measurement, has the same direction as $\boldsymbol{\alpha}$, the effect of effort on the principal's benefit. Actually, given that Assumption 4 holds, similar to the case of balanced tasks in the baseline model, separating the tasks does not strictly improve the performance induced by implicit incentives. In such cases, task bundling is at least weakly preferred by patient parties. However, as shown in the previous propositions, when the parties become impatient so that no implicit incentives are available, task separation makes explicit incentives more effective and is better.

There is a remark on the result in Schöttner (2008) that the benefit of task bundling is not necessarily caused by strong retaliation for reneging on informal bonuses. As mentioned above, in addition to the assumption that the principal's benefit is observable, Schöttner (2008) restricts the model to three tasks in total, which implies that an agent performs only a single tasks under task separation. Since there exists a verifiable performance measurement, the principal can provide the singletasking agent with the first best incentive only through a formal contract and then a discretionary bonus is paid only to the other agent. Consequently, the principal's problem under task separation is essentially interpreted as designing explicit and implicit incentives for a single agent with two tasks. Then the interpretation of the trade-off in her model is different from our insight: task separation mitigates the multitasking incentive problem through reducing the assigned tasks, but at the same time improves the value of spot contracting, which weakens the punishment for reneging on discretionary bonuses. This scenario corresponds to our setup where $\boldsymbol{\alpha}_i$ and $\boldsymbol{\eta}_i$ are linearly dependent for some i: given linear dependence of $\boldsymbol{\alpha}_i$ and $\boldsymbol{\eta}_i$, agent i's effort can be at the first best level only through a formal contract. If, in addition, the value of spot contracting is strictly lower under task bundling (i.e. (12) holds), then for intermediate discount rate, task bundling is strictly better. However, as Proposition 9 states, the optimality of task bundling for patient parties holds even if (12) is violated, i.e. the value of spot contracting is the same regardless of job design. This is because, as demonstrated in our baseline model, dispersion of discretionary bonuses between agents under task separation tightens the dynamic enforcement constraint. Specifically, provided that $\boldsymbol{\alpha}_i$ and $\boldsymbol{\eta}_i$ are linearly independent for both $i = 1, 2$, explicit incentives cannot perfectly resolve the incentive problem even under task separation so that the principal needs to provide implicit incentives for both agents to improve efficiency. However, the assumption that $\boldsymbol{\alpha}$ and $\boldsymbol{\mu}$ are linearly dependent implies that, as in the case of perfectly balanced tasks in our baseline model, the desirable effort pair can be implemented with a smaller informal bonus under task bundling than task separation. Therefore, the optimality of task bundling for patient parties can be attributed to reduction of informal bonuses under task bundling as well as strengthening the retaliation for reneging.

Finally, we should mention another remark that although we verify the argument that task bundling is better for patient parties under a weaker assumption than Schöttner (2008), the assumption still seems to be restrictive. Specifically, the argument crucially depends on the assumption on the performance measurements that assures that task separation and task bundling are indifferent for patient parties. In general, as long as the nonverifiable performance measurement is distorted from the true performance even slightly, task separation generically allows strictly better incentive provision *via* informal bonuses than task bundling. Consequently, as argued in Proposition 7, task bundling is not chosen by patient parties.

6 Conclusion

Recent productive organizations do not necessarily take a homogeneous form of job design: in some firms, broader task assignment is prevalent while division of labour supports high productivity in other firms. In order to clarify the difference of the optimality of task assignment from incentive perspectives, we investigate the optimal job design in multitasking environments where relational incentives are necessary.

In the baseline model where there is no verifiable performance measurement, we find an intuitive trade-off between misallocation of effort and dispersion of informal bonuses. Task separation can potentially mitigate misallocation of effort. However since the principal must pay informal bonuses to all the parties, task separation makes it harder to establish credibility of honouring the informal bonuses. Consequently, a clear-cut relationship emerges: task separation is preferred if and only if the parties are patient. The degree of misallocation of effort under task bundling relative to task separation in general depends on how the performance measurement is distorted from the principal's benefit. When the degree of misallocation of effort is sufficiently high, which is called strongly unbalanced tasks in the model, the principal may exclude some tasks to focus on other important tasks. In the extended model, where there is a verifiable performance measurement, task separation also affects the incentive provision *via* explicit incentives as well as informal bonuses. As a result, the optimal job design is not necessarily monotonic with respect to the parties' patience. In general, task bundling is optimal only when the parties' patience is intermediate.

There may be various ways of extension and application of job design problem in relational contracts. Theoretically, it is possible to consider generalization of more agents and/or performance measurements. We guess that this extension makes the analysis substantially complicated but finds few additional insights. One may be interested in comparison of several patterns of task assignment under task separation. Since we successfully characterize the optimal contract under task separation as a closed-form solution, two different modes of task assignment under task separation can be compared at least in a computational way. Finally, we totally abstract technological effects of changing job design. To disentangle and quantify the incentive effect, which we pointed out, and other technological effects, which we abstract, of job design would be an important empirical question.

A Appendix: Definitions and Proofs

A.1 Definition of Strategies and Equilibrium

We provide definitions of strategies and equilibria in the baseline model. They are similarly defined in the extended model of Section 5.

Given job design, task bundling or task separation, a strategy in the repeated game is defined as follows. Denote a profile of observable events (or simply events) in period τ by h_τ, which consists of: (i) lump-sum transfers (w_τ^1, w_τ^2) specified by a formal contract;

(ii) agent i's decision on whether to accept or reject the contract, $q^i_\tau \in \{ac, re\}$; and (iii) given $(q^1_\tau, q^2_\tau) = (ac, ac)$, a performance measurement $x_\tau \in \{S, F\}$ and discretionary bonuses paid to the agents, $(b_{1\tau}, b_{2\tau})$. Denote the set of all possible events by $\mathfrak{H}$. A public history up to period τ is defined as $h^\tau \in \mathfrak{H}^{\tau-1}$ for $\tau \geq 1$ and $h^0 \equiv \emptyset$. A public strategy of the principal is a function of public history, $\sigma^P(h^\tau)$, which specifies after h^τ: (i) lump-sum transfers $(w_{1\tau}, w_{2\tau})$ offered to the agents; and (ii) discretionary bonuses paid to agent i for each $i = 1, 2$ given a formal contract and a performance measurement x_t. A public strategy of agent i in period τ is also a function of public history, $\sigma^i(h^\tau)$, which specifies after h^τ: (i) a decision of acceptance or rejection to a formal contract; and (ii) effort e_{nt} on each assigned task given that a formal contract was accepted. A strategy profile $(\sigma^P, \sigma^1, \sigma^2)$ is a perfect public equilibrium if all σ^P, σ^1, and σ^2 are public strategies and the strategy profile with a Bayesian consistent belief is a sequential equilibrium.

A public strategy profile is stationary if (i) on the equilibrium path, offered lump-sum transfers, the agents' decisions, and discretionary bonuses are independent of time t; and (ii) the players revert to a static Nash equilibrium if and only if the players have observed a deviation at an information set on which the principal chooses discretionary bonus payments.

A.2 Proof of Proposition 1

The bonus of the optimal relational contract solves problem (4). If the constraint is ignored, then the first order condition implies that the solution is $\overline{\beta} = A/M$. This solution satisfies the constraint with strict inequality if and only if

$$\frac{1}{r}\frac{A^2}{2M} > \frac{A}{M} \iff r < \frac{A}{2}.$$

In this case, the value of the objective is $A^2/2M$, which is independent of r.

Otherwise the constraint must be binding, which implies that $\overline{\beta}$ is either $2(A-r)/M$ or 0. Since the constraint is binding, the objective function satisfies $A\overline{\beta} - M\overline{\beta}^2/2 = r\overline{\beta}$, implying that $\overline{\beta}$ should be higher. Then for $r < A$, the solution satisfies $\overline{\beta} = 2(A-r)/M$ and the value of the objective is $2r(A-r)/M$. Since the derivative of the value with respect to r is $2(A-2r)/M$, the value $2r(A-r)/M$ is decreasing in $r \in (A/2, A)$. On the other hand, for $r \geq A$, the solution satisfies $\overline{\beta} = 0$.

Therefore, the solution $\overline{\beta}$ and the principal's payoff Y^B are summarized as follows:

$$(\overline{\beta}, Y_B) = \begin{cases} \left(\dfrac{A}{M}, \dfrac{A^2}{2M}\right) & \text{if } r < \dfrac{A}{2}, \\ \left(\dfrac{2(A-r)}{M}, \dfrac{2r(A-r)}{M}\right) & \text{if } \dfrac{A}{2} \leq r < A, \\ (0,0) & \text{if } r \geq A. \end{cases}$$

Y^B is continuous in r due to the theorem of the maximum. The dynamic enforcement constraint is binding if and only if $r \geq A/2$.

A.3 Proof of Proposition 2

Similar to Proposition 1, the bonus of the optimal relational contract solves problem (8). If the constraints are ignored, then the first order condition implies that the solution is $\beta_i = a_i/m_i$ for each $i = 1, 2$. This solution satisfies the constraint if and only if

$$\frac{1}{2r}\sum_{i=1}^{2}\frac{a_i^2}{m_i} > \sum_{i=1}^{2}\frac{a_i}{m_i} \iff r < \frac{\sum_{i=1}^{2}(a_i^2/m_i)}{2\sum_{i=1}^{2}(a_i/m_i)} = \frac{a_1^2 m_2 + a_2^2 m_1}{2(a_1 m_2 + a_2 m_1)}. \tag{13}$$

In this case, the value of the objective is $\sum_{i=1}^{2}(a_i^2/m_i)/2$, which is independent of r.

Otherwise at least either the dynamic enforcement constraint or the non-negative constraint of β_i must be binding. To investigate which constraint is binding, define the Lagrangian as

$$\mathcal{L} = \sum_{i=1}^{2}\left(a_i\beta_i - \frac{m_i}{2}\beta_i^2\right) + \lambda\left[\frac{1}{r}\sum_{i=1}^{2}\left(a_i\beta_i - \frac{m_i}{2}\beta_i^2\right) - \sum_{i=1}^{2}\beta_i\right] + \sum_{i=1}^{2}\kappa_i\beta_i,$$

where λ is the multiplier for the dynamic enforcement constraint and κ_i is the multiplier for the non-negative constraint. The first order condition with respect to β_i is

$$\frac{\partial\mathcal{L}}{\partial\beta_i} = \left(1 + \frac{\lambda}{r}\right)(a_i - m_i\beta_i) - \lambda + \kappa_i = 0 \iff \beta_i = \frac{a_i}{m_i} - \frac{\lambda - \kappa_i}{m_i(1 - \lambda/r)}.$$

If $\lambda = 0$ and $\kappa_i = 0$ for all $i = 1, 2$, then $\beta_i = a_i/m_i$ for $i = 1, 2$ and as seen above, this satisfies the constraint if and only if (13) is satisfied with weak inequality. If $\lambda = 0$ and $\kappa_i > 0$ for some $i = 1, 2$, then the first order condition implies $\beta_i = (a_i + \kappa_i)/m_i > 0$ whereas the complementary slackness condition implies $\beta_i = 0$, a contradiction. Therefore, in the rest of the cases, suppose $\lambda > 0$, which implies that the dynamic enforcement constraint is binding by the complementary slackness condition.

First suppose that $\kappa_1 = \kappa_2 = 0$. Then combining the first order condition for both $i = 1,2$ yields $a_1 - m_1\beta_1 = a_2 = m_2\beta_2$. Plugging this equation into the binding dynamic enforcement constraint eliminates β_2 as follows:

$$\frac{1}{r}\left[a_1\beta_1 - \frac{m_1}{2}\beta_1^2 + \frac{a_2 - a_1 + m_1\beta_1}{m_2}\left(a_2 - \frac{m_2}{2}\frac{a_2 - a_1 + m_1\beta_1}{m_2}\right)\right] = \beta_1 + \frac{a_2 - a_1 + m_1\beta_1}{m_2}$$

$$\iff \quad \beta_1 = \frac{a_1 - r}{m_1} \pm \sqrt{\left(\frac{a_1 - r}{m_1}\right)^2 + \frac{(a_2 - a_1)(A - 2r)}{m_1 M}} = \frac{a_1 - r \pm K(r)}{m_1},$$

where $K(r) = \sqrt{[(m_2a_1 + m_1a_2)/M - r]^2 + m_1m_2(a_1 - a_2)^2/M^2}$. Note that $K(r)$ is real and non-negative for any r. Plugging this β_1 yields

$$(\beta_1, \beta_2) = \left(\beta_1, \frac{a_2 - a_1 + m_1\beta_1}{m_2}\right) = \left(\frac{a_1 - r + K(r)}{m_1}, \frac{a_2 - r + K(r)}{m_2}\right), \left(\frac{a_1 - r - K(r)}{m_1}, \frac{a_2 - r - K(r)}{m_2}\right).$$

Recall that the dynamic enforcement constraint is binding. Then the objective function satisfies $\sum_{i=1}^{2}(a_i\beta_i - m_i\beta_i^2/2) = r\sum_{i=1}^{2}\beta_i$, implying that β_i should be higher. Then the optimal solution should be $\beta_i = (a_i - r + K(r))/m_i$ for each $i = 1,2$, which yields the value $r\sum_{i=1}^{2}(a_i - r + K(r))/m_i$. We now check the condition under which $\beta_i \geq 0$ for $i = 1,2$. Since $a_1 \geq a_2$ by Assumption 1, for $r < a_2$, β_i is positive and the non-negative constraint is satisfied for both $i = 1,2$. For $r \geq a_2$, there are two sub cases to be considered.

Case of $a_1 = a_2$: Since $r \geq a_2 = A/2$, $K(r) = \sqrt{(A/2 - r)^2} = r - A/2$. Then $\beta_i = [K(r) - r + A/2]/m_i = 0$ for any $r \geq A/2$.

Case of $a_1 > a_2$: Obviously, β_1 is positive for $r < a_1$. For $r \geq a_1$, $\beta_1 > 0$ is satisfied if and only if

$$K(r) > r - a_1 \quad \iff \quad \left(\frac{m_2a_1 + m_1a_2}{M} - r\right)^2 + \frac{m_1m_2(a_1 - a_2)^2}{M^2} > (r - a_1)^2$$
$$\iff \quad r > \frac{A}{2},$$

which holds given $r \geq a_1$ since $a_1 > A/2$. Then β_1 is positive for any $r \geq a_2$. Given $r \geq a_2$, β_2 is non-negative if and only if

$$K(r) \geq r - a_2 \quad \iff \quad \left(\frac{m_2a_1 + m_1a_2}{M} - r\right)^2 + \frac{m_1m_2(a_1 - a_2)^2}{M^2} \geq (r - a_2)^2$$
$$\iff \quad r \leq \frac{A}{2}.$$

Therefore $\beta_i \geq 0$ for both $i = 1, 2$ if and only if $r \leq A/2$. By replacing weak inequalities with strict inequalities, we also see that $\beta_i > 0$ for both $i = 1, 2$ if and only if $r < A/2$.

As the non-negative constraint is innocuous when $a_1 = a_2$, in the rest of the proof suppose $a_1 > a_2$ and $r \geq A/2$. When $\lambda > 0$ and $\kappa_1 > 0$, the complementary slackness condition implies $\beta_1 = 0$. Then the binding dynamic enforcement constraint implies

$$\frac{1}{r}\left(a_2\beta_2 - \frac{m_2}{2}\beta_2^2\right) = \beta_2 \iff \beta_2 = 0, \frac{2(a_2 - r)}{m_2}.$$

Due to the non-negative constraint on β_2 and $r \geq A/2 > a_2$, β_2 must be zero and as a result the value of the objective function is also zero. On the other hand, if $\lambda > 0$ and $\kappa_2 > 0$, then the complementary slackness condition implies $\beta_2 = 0$. Then the binding dynamic enforcement constraint implies

$$\frac{1}{r}\left(a_1\beta_1 - \frac{m_1}{2}\beta_1^2\right) = \beta_1 \iff \beta_1 = 0, \frac{2(a_1 - r)}{m_1}.$$

Similar to the previous argument, β_1 should be higher in order to increase the value of the objective function. For $r \in [A/2, a_1)$, a positive bonus $\beta_1 = 2(a_1 - r)/m_1$ is feasible and yields a higher value $2r(a_1 - r)/m_1$ than zero. Then the solution satisfies $\kappa_2 > 0$. On the other hand, for $r \geq a_1$, the bonus for agent 1 is also zero, yielding the value equal to zero.

Therefore, the solution (β_1, β_2) is summarized as

$$(\beta_1, \beta_2) = \begin{cases} \left(\dfrac{a_1}{m_1}, \dfrac{a_2}{m_2}\right) & \text{if } r < \dfrac{(a_1^2 m_2 + a_2^2 m_1)}{2(a_1 m_2 + a_2 m_1)}, \\ \left(\dfrac{a_1 - r + K(r)}{m_1}, \dfrac{a_2 - r + K(r)}{m_2}\right) & \text{if } \dfrac{(a_1^2 m_2 + a_2^2 m_1)}{2(a_1 m_2 + a_2 m_1)} \leq r < \dfrac{A}{2}, \\ \left(\dfrac{2(a_1 - r)}{m_1}, 0\right) & \text{if } \dfrac{A}{2} \leq r < a_1, \\ (0, 0) & \text{if } r \geq a_1, \end{cases}$$

and the principal's payoff Y^S is

$$Y^S = \begin{cases} \dfrac{1}{2}\displaystyle\sum_{i=1}^{2} \dfrac{a_i^2}{m_i} & \text{if } r < \dfrac{(a_1^2 m_2 + a_2^2 m_1)}{2(a_1 m_2 + a_2 m_1)}, \\ r\displaystyle\sum_{i=1}^{2} \dfrac{a_i - r + K(r)}{m_i} & \text{if } \dfrac{(a_1^2 m_2 + a_2^2 m_1)}{2(a_1 m_2 + a_2 m_1)} \leq r < \dfrac{A}{2}, \\ \dfrac{2r(a_1 - r)}{m_1} & \text{if } \dfrac{A}{2} \leq r < a_1, \\ 0 & \text{if } r \geq a_1. \end{cases}$$

Y^S is continuous in r due to the theorem of the maximum. For $r \in ((a_1^2 m_2 + a_2^2 m_1)/2(a_1 m_2 + a_2 m_1), a_1)$, since $Y^S > 0$ and $\lambda > 0$, the envelope theorem implies

$$\frac{\partial Y^S}{\partial r} = \frac{\partial \mathcal{L}}{\partial r} = -\frac{\lambda}{r^2} Y^S < 0,$$

meaning that Y^S is decreasing in $r \in ((a_1^2 m_2 + a_2^2 m_1)/2(a_1 m_2 + a_2 m_1), a_1)$.

A.4 Proof of Proposition 3

When $a_1/a_2 = m_1/m_2$, we have $a_1/m_1 = a_2/m_2 = A/M$. Then, from Proposition 2, the principal's payoff under task separation is written as

$$Y^S = \begin{cases} \dfrac{A^2}{2M} & \text{if } r < \dfrac{A}{4}, \\ \dfrac{2Ar}{M} + \dfrac{Mr[K(r) - r]}{m_1 m_2} & \text{if } \dfrac{A}{4} \leq r < \dfrac{A}{2}, \\ \dfrac{2r}{M}\left(A - \dfrac{Mr}{m_1}\right) & \text{if } \dfrac{A}{2} \leq r < a_1, \\ 0 & \text{if } r \geq a_1. \end{cases}$$

For $r \leq A/4$, Y^S is equal to Y^B. For $r \in (A/4, A/2)$, since Y^S is decreasing in r, Y^S is strictly less than $A^2/2M = Y^B$. For $r \in [A/2, a_1)$,

$$Y^B - Y^S = \frac{2r}{M}\left(\frac{M}{m_1} - 1\right) r > 0.$$

For $r \in [a_1, A)$, Y^B is positive while Y^S is zero, which implies $Y^B > Y^S$. Finally, for $r \geq A$, $Y^B = Y^S = 0$.

A.5 Proof of Proposition 4

First note that $(m_1 a_2^2 + m_2 a_1^2)/(m_1 a_2 + m_2 a_1) < A$. Then, for $r \leq (m_1 a_2^2 + m_2 a_1^2)/2(m_1 a_2 + m_2 a_1)$,

$$Y^S - Y^B = \frac{1}{2}\left(\sum_{i=1}^{2} \frac{a_i^2}{m_i} - \frac{A^2}{M}\right) = \frac{(a_1 m_2 - a_2 m_1)^2}{2 m_1 m_2 M},$$

which is strictly positive since $a_1/a_2 \neq m_1/m_2$. Therefore $Y^S > Y^B$.

For $r \in ((m_1 a_2^2 + m_2 a_1^2)/2(m_1 a_2 + m_2 a_1), A/2]$, the following claim is useful for the result.

Claim 1 *Under Assumption 1, $Y^S \leq Y^B$ at $r = A/2$ if and only if $a_1/a_2 \leq 2m_1/m_2 + 1$.*

Proof (Claim 1) *From Propositions 1 and 2, $Y^S \leq Y^B$ at $r = A/2$ if and only if*

$$\frac{A\left[a_1m_2 + a_2m_1 - M[A/2 - K(A/2)]\right]}{2m_1m_2} \leq \frac{A^2}{2M}$$
$$\iff \quad K\left(\frac{A}{2}\right) \leq \frac{Am_1m_2}{M^2} + \frac{A}{2} - \frac{a_1m_2 + a_2m_1}{M},$$

which is satisfied only if the right hand side is non-negative. Given that the right hand side is non-negative, this inequality is satisfied if and only if

$$\left(\frac{m_2a_1 + m_1a_2}{M} - \frac{A}{2}\right)^2 + \frac{m_1m_2(a_1 - a_2)^2}{M^2} \leq \left(\frac{Am_1m_2}{M^2} + \frac{A}{2} - \frac{m_2a_1 + m_1a_2}{M}\right)^2$$
$$\iff \left(\frac{a_1}{a_2}\frac{m_1}{m_2} - \frac{m_1}{m_2} + 2\frac{a_1}{a_2}\right)\left(\frac{a_1}{a_2} - 2\frac{m_1}{m_2} - 1\right) \leq 0.$$

Since $a_1/a_2 \geq 1$ implies that the term in the first parentheses is positive, this is satisfied if and only if $a_1/a_2 \leq 2m_1/m_2 + 1$. Finally, given $a_1/a_2 \leq 2m_1/m_2 + 1$,

$$\begin{aligned}
& \frac{Am_1m_2}{M^2} + \frac{A}{2} - \frac{m_2a_1 + m_1a_2}{M} \\
= \ & \frac{m_2^2a_2}{2M^2}\left[\left(\frac{a_1}{a_2} - 1\right)\left(\frac{m_1}{m_2}\right)^2 + 2\left(\frac{a_1}{a_2} + 1\right)\frac{m_1}{m_2} - \left(\frac{a_1}{a_2} - 1\right)\right] \\
\geq \ & \frac{m_2^2a_2}{2M^2}\left[\left(\frac{a_1}{a_2} - 1\right)\left(\frac{a_1}{2a_2} - \frac{1}{2}\right)^2 + 2\left(\frac{a_1}{a_2} + 1\right)\left(\frac{a_1}{2a_2} - \frac{1}{2}\right) - \left(\frac{a_1}{a_2} - 1\right)\right] \\
= \ & \frac{m_2^2a_2}{2M^2}\left(\frac{a_1}{a_2} - 1\right)\left[\frac{1}{4}\left(\frac{a_1}{a_2} - 1\right) + \left(\frac{a_1}{a_2} + 1\right) - 1\right] \\
= \ & \frac{m_2^2a_2}{2M^2}\left(\frac{a_1}{a_2} - 1\right)\left[\frac{1}{4}\left(\frac{a_1}{a_2} - 1\right) + \frac{a_1}{a_2}\right] \geq 0,
\end{aligned}$$

where the inequalities are due to $m_1/m_2 \geq (a_1/a_2 - 1)/2$ and $a_1/a_2 \geq 1$. □

From Claim 1, $Y^S \leq Y^B$ at $r = A/2$ when the tasks are weakly unbalanced. Recall that from Propositions 1 and 2, for $r \in ((m_1a_2^2 + m_2a_1^2)/2(m_1a_2 + m_2a_1), A/2]$, Y^B is constant while Y^S is continuous and decreasing in r. Then there uniquely exists $\tilde{r} \in ((m_1a_2^2 + m_2a_1^2)/2(m_1a_2 + m_2a_1), A/2]$ such that $Y^S \gtreqless Y^B$ if and only if $r \lesseqgtr \tilde{r}$. Such $\tilde{r}$ must satisfy

$$\frac{\tilde{r}\left[(a_1m_2 + a_2m_1) - M[\tilde{r} - K(\tilde{r})]\right]}{m_1m_2} = \frac{A^2}{2M} \iff K(\tilde{r}) = \frac{m_1m_2A^2}{2M^2\tilde{r}} - \frac{a_1m_2 + a_2m_1}{M} + \tilde{r},$$

which implies

$$K(\tilde{r})^2 = \left(\frac{m_1m_2A^2}{2M^2\tilde{r}} - \frac{a_1m_2 + a_2m_1}{M} + \tilde{r}\right)^2 \iff \tilde{r} = \frac{A^2m_1}{4Ma_1}, \frac{A^2m_2}{4Ma_2}.$$

Let $k, \ell \in \{1,2\}$ satisfy $m_\ell/a_\ell < m_k/a_k$ and suppose $\tilde{r} = A^2 m_\ell / 4Ma_\ell$.[17] Then

$$\begin{aligned}\frac{m_1 a_2^2 + m_2 a_1^2}{m_1 a_2 + m_2 a_1} - \frac{A^2 m_\ell}{2Ma_\ell} &= \frac{2Ma_\ell[m_k a_\ell^2 + m_\ell a_k^2] - A^2 m_\ell (m_k a_\ell + m_\ell a_k)}{2Ma_\ell(m_1 a_2 + m_2 a_1)} \\ &= \frac{2a_k a_\ell (m_k/a_k - m_\ell/a_\ell)(m_\ell a_\ell^2 + m_\ell a_k^2 + 2m_k a_\ell^2)}{2Ma_\ell(m_1 a_2 + m_2 a_1)} > 0,\end{aligned}$$

which contradicts $\tilde{r} \in ((m_1 a_2^2 + m_2 a_1^2)/2(m_1 a_2 + m_2 a_1), A/2]$. Therefore $\tilde{r} = \max\{m_1/a_1, m_2/a_2\} A^2/4M$.

For $r \in (A/2, a_1]$,

$$\begin{aligned}Y^S - Y^B &= \frac{2r}{m_1 M}\left[(a_1 m_2 - a_2 m_1) - m_2 r\right] \\ &< \frac{2r}{m_1 M}\left[(a_1 m_2 - a_2 m_1) - m_2 \frac{A}{2}\right] = \frac{r}{m_1 m_2 a_2 M}\left(\frac{a_1}{a_2} - 2\frac{m_1}{m_2} - 1\right) \le 0,\end{aligned}$$

where the last inequality is due to that the tasks are weakly unbalanced.

For $r \in [a_1, A)$, Y^B is positive while Y^S is zero, which implies $Y^B > Y^S$. Finally, for $r \ge A$, $Y^B = Y^S = 0$.

A.6 Proof of Proposition 5

For $r \le (m_1 a_2^2 + m_2 a_1^2)/2(m_1 a_2 + m_2 a_1)$, we have $Y^S > Y^B$ due to the same argument as Proposition 4.

For $r \in ((m_1 a_2^2 + m_2 a_1^2)/2(m_1 a_2 + m_2 a_1), A/2]$, Claim 1 implies that $Y^S > Y^B$ at $r = A/2$ when the tasks are strongly unbalanced. Since Y^B is constant in r while Y^S is continuous and decreasing in r, we obtain $Y^S > Y^B$ for all $r \in ((m_1 a_2^2 + m_2 a_1^2)/2(m_1 a_2 + m_2 a_1), A/2]$.

For $r \in (A/2, a_1]$, since

$$Y^S - Y^B = \frac{2r}{m_1 m_2 M}\left(a_1 - a_2 \frac{m_1}{m_2} - r\right),$$

$Y^S \gtreqless Y^B$ if and only if $r \lesseqgtr a_1 - a_2 m_1/m_2$. Note that obviously $a_1 > a_1 - a_2 m_1/m_2$ and

$$a_1 - a_2\frac{m_1}{m_2} - \frac{A}{2} = \frac{a_2}{2}\left(\frac{a_1}{a_2} - 2\frac{m_1}{m_2} - 1\right) > 0,$$

where the inequality is satisfied since the tasks are strongly unbalanced. Then $a_1 - a_2 m_1/m_2 \in (A/2, a_1]$. Note furthermore that whenever $Y^S > Y^B$, tasks $n \in \mathcal{N}_2$ are excluded due to Proposition 2.

The rest of the proof is the same as Proposition 4: for $Y^B > Y^S = 0$ for $r \in [a_1, A)$ and $Y^B = Y^S = 0$ for $r \ge A$.

[17] Note that $m_1/a_1 \ne m_2/a_2$ since the tasks are weakly unbalanced.

A.7 Proof of Lemma 3

Since $r \in [A/2, a_1 - a_2 m_1/m_2)$, the proof of Propositions 1 and 2 implies the the optimal bonus satisfies $\overline{\beta} = 2(A-r)/M$ and $\beta_1 = 2(a_1 - r)/m_1$. Then

$$\overline{\beta} - \beta_1 = \frac{2}{Mm_1}(Am_1 - a_1 M + m_2 r) = \frac{2m_2}{Mm_1}\left(\frac{a_2 m_1}{m_2} - a_1 + r\right)$$

and

$$a_2\overline{\beta} - \frac{m_2\overline{\beta}^2}{2} = \frac{2(A-r)m_2}{M^2}\left(a_2\frac{m_1}{m_2} - a_1 + r\right).$$

Both are negative since $r < a_1 - a_2 m_1/m_2 < A$.

A.8 Proof of Proposition 6

Let $\hat{r}(\rho_{A1})$ be defined as (9). It is easy to check that $\hat{r}(\rho_{A1})$ is continuous in ρ_{A1}. Furthermore Propositions 3, 4, and 5 imply that $Y^S > Y^B$ if $r < \hat{r}(\rho_{A1})$ and the tasks are not perfectly balanced (i.e. $\rho_{A1} \neq \rho_{M1}$); and $Y^S < Y^B$ if $r \in (\hat{r}(\rho_{A1}), A)$.

1. Suppose first $\rho_{M1} < 1/2$. Since $\rho_{M1} < 1/2 \leq \rho_{A1}$, $\max\{\rho_{M1}/\rho_{A1}, (1-\rho_{M1})/(1-\rho_{A1})\} = (1-\rho_{M1})/(1-\rho_{A1})$. Then $\hat{r}(\rho_{A1})$ is increasing in ρ_{A1}. Furthermore, since $\rho_{A1} \in [1/2, 1)$, $\hat{r}(\rho_{A1}) \in [(1-\rho_{M1})A/2, A)$.

   (a) Suppose $r \leq (1-\rho_{M1})A/2$. Then $r \leq \hat{r}(\rho_{A1})$ for any $\rho_{A1} \in [1/2, 1)$.

   (b) Suppose $r \in ((1-\rho_{M1})A/2, A)$. Let $\hat{\rho}_{A1} \equiv \hat{r}^{-1}(r) \in (1/2, 1)$. Then for $\rho_{A1} > \hat{\rho}_{A1}$, we obtain $r < \hat{r}(\rho_{A1})$, implying $Y^S > Y^B$. Similarly, for $\rho_{A1} < \hat{\rho}_{A1}$, since $r > \hat{r}(\rho_{A1})$, $Y^S < Y^B$.

2. Suppose next $\rho_{M1} > 1/2$. Then $\hat{r}(\rho_{A1})$ is expressed as

$$\hat{r}(\rho_{A1}) = \begin{cases} \dfrac{\rho_{M1}A}{4\rho_{A1}} & \text{if } \rho_{A1} \in \left[\dfrac{1}{2}, \rho_{M1}\right], \\ \dfrac{(1-\rho_{M1})A}{4(1-\rho_{A1})} & \text{if } \rho_{A1} \in \left[\rho_{M1}, \dfrac{1+\rho_{M1}}{2}\right], \\ \dfrac{A(\rho_{A1}-\rho_{M1})}{1-\rho_{M1}} & \text{if } \rho_{A1} \in \left(\dfrac{1+\rho_{M1}}{2}, 1\right), \end{cases}$$

   which is decreasing in $\rho_{A1} \in [1/2, \rho_{M1})$ and increasing in $\rho_{A1} \in (\rho_{M1}, 1)$. Note that $\hat{r}(1/2) = \rho_{M1}A/2$, $\hat{r}(\rho_{M1}) = A/4$, and $\lim_{\rho_{A1}\to 1}\hat{r}(\rho_{A1}) = A > \hat{r}(1/2)$.

   (a) Suppose $r \leq A/4$. Since $r < \hat{r}(\rho_{A1})$ for any $\rho_{A1} \in [1/2, 1) \setminus \{\rho_{M1}\}$, Propositions 4 and 5 imply $Y^S > Y^B$.

(b) Suppose $r \in (A/4, \rho_{M1}A/2]$. Then there exactly exist two values, $\underline{\rho}_{A1} \in [1/2, \rho_{M1})$ and $\overline{\rho}_{A1} \in (\rho_{M1}, 1)$ such that $\hat{r}(\rho_{A1}) = r$ for $\rho_{A1} = \underline{\rho}_{A1}, \overline{\rho}_{A1}$. By the shape of $\hat{r}(\rho_{A1})$, $r < \hat{r}(\rho_{A1})$ if and only if $\rho_{A1} < \underline{\rho}_{A1}$ or $\rho_{A1} > \overline{\rho}_{A1}$, in which $Y^S > Y^B$. Similarly, we see that $r > \hat{r}(\rho_{A1})$ if and only if $\underline{\rho}_{A1} < \rho_{A1} < \overline{\rho}_{A1}$, in which $Y^S < Y^B$.

(c) Suppose $r \in (\rho_{M1}A/2, A)$. Then the shape of $\hat{r}(\rho_{A1})$ implies that there uniquely exists $\hat{\rho}_{A1} \equiv \hat{r}^{-1}(r) \in (\rho_{M1}, 1)$ and for $\rho_{A1} > \hat{\rho}_{A1}$, we obtain $r < \hat{r}(\rho_{A1})$, implying that $Y^S > Y^B$. Similarly, for $\rho_{A1} < \hat{\rho}_{A1}$, since $r > \hat{r}(\rho_{A1})$, $Y^S < Y^B$.

3. For $r \in [A/2, A)$, if $\rho_{A1} > \hat{\rho}_{A1}$, then since $Y^S > Y^B$, Proposition 2 implies that tasks $n \in \mathcal{N}_2$ are excluded.

B Appendix: Analysis of Section 5

This section provides an analysis of the extended model in Section 5.

B.1 Proof of Lemma 5

Consider first spot contracting under task bundling and suppose that a formal contract $(\overline{w}(S), \overline{w}(F))$ is accepted. Then, as in the standard moral hazard problem, the formal contract is a solution to the following constrained optimization problem:

$$
\begin{aligned}
\max_{\Delta\overline{w}, \overline{w}(F)} \quad & f(\boldsymbol{e}) - h(\boldsymbol{e})\Delta\overline{w} - \overline{w}(F) \\
\text{subject to} \quad & \boldsymbol{e} \in \arg\max_{\tilde{\boldsymbol{e}} \in \mathbb{R}_+^N} \left[h(\tilde{\boldsymbol{e}})\Delta\overline{w} + \overline{w}(F) - \sum_{n \in \mathcal{N}} c_n(\tilde{e}_n) \right], \\
& h(\boldsymbol{e})\Delta\overline{w} + \overline{w}(F) - \sum_{n \in \mathcal{N}} c_n(e_n) \geq 0,
\end{aligned}
$$

where $\Delta\overline{w} \equiv \overline{w}(S) - \overline{w}(F)$. The first constraint, the incentive compatibility constraint, can be replaced with the first order condition as $e_n = \Delta\overline{w}\eta_n/\gamma_n$. Moreover, $\overline{w}(F)$ is pinned down such that the second constraint, the participation constraint, is satisfied with equality. Then plugging these transforms the objective function into $\boldsymbol{\alpha}\Gamma^{-1}\boldsymbol{\eta}'\Delta\overline{w} - \boldsymbol{\eta}\Gamma^{-1}\boldsymbol{\eta}'(\Delta\overline{w})^2/2$. The first order condition implies that the optimal $\Delta\overline{w}$ satisfies $\Delta\overline{w} = \boldsymbol{\alpha}\Gamma^{-1}\boldsymbol{\eta}'/\boldsymbol{\eta}\Gamma^{-1}\boldsymbol{\eta}'$ and the optimized value is $\underline{y}^B$.

Under task separation, the optimal spot contract solves the following problem:

$$\begin{aligned}
\max_{\Delta w_1, w_1(F), \Delta w_2, w_2(F)} \quad & f(\boldsymbol{e}) - h(\boldsymbol{e}) \sum_{i=1}^{2} \Delta_i w_i - \sum_{i=1}^{2} w_i(F) \\
\text{subject to} \quad & \boldsymbol{e}_i \in \arg\max_{\tilde{\boldsymbol{e}}_i \in \mathbb{R}_+^{\# N_i}} \left[h(\tilde{e}_i, \boldsymbol{e}_{-i}) \Delta w_i + w_i(F) - \sum_{n \in \mathcal{N}_i} c_n(\tilde{e}_n^i) \right], \forall i = 1, 2, \\
& h(\boldsymbol{e}) \Delta w_i + w_i(F) - \sum_{n \in \mathcal{N}_i} c_n(e_n) \geq 0, \forall i = 1, 2.
\end{aligned}$$

The similar procedure yields the optimal spot contract that satisfies $\Delta w_i = \boldsymbol{\alpha}_i \Gamma^{-1} \boldsymbol{\eta}_i' / \boldsymbol{\eta}_i \Gamma^{-1} \boldsymbol{\eta}_i'$ and the optimized value $\underline{y}^S$.

Finally, it is easy to obtain

$$\underline{y}^S - \underline{y}^B = \frac{(\boldsymbol{\alpha}_1 \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_1' \boldsymbol{\eta}_2 \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_2' - \boldsymbol{\alpha}_2 \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_2' \boldsymbol{\eta}_1 \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_1')^2}{2 \boldsymbol{\eta}_1 \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_1' \boldsymbol{\eta}_2 \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_2' \boldsymbol{\eta} \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}'} \geq 0,$$

where the inequality holds with strict inequality if and only if (12) is satisfied.

B.2 Optimal Relational Contracts

B.2.1 Optimization Problems

The optimal relational contract can similarly be characterized by an optimization problem.

Lemma 6 *Suppose that there is a verifiable performance measurement z_t and Assumption 2 holds. Suppose in addition that the optimal job design is task bundling. Let $\boldsymbol{e}$ be the effort and $(\overline{\beta}(S), \overline{\beta}(F), \overline{w}(S), \overline{w}(F))$ be the pair of the monetary transfer paid to agent 1 of the optimal relational contract. Then they solve the following optimization problem:*

$$\begin{aligned}
\max_{\overline{\beta} \geq 0, e, \Delta \overline{w}} \quad & Y(\boldsymbol{e}) \\
\textit{subject to} \quad & e \in \arg\max_{\boldsymbol{e}' \in \mathbb{R}_+^N} \left[\overline{\beta} p(\boldsymbol{e}') + \Delta \overline{w} h(\boldsymbol{e}') - \sum_{n \in \mathcal{N}} c_n(e_n') \right], & (14) \\
& -\overline{\beta} + \frac{\delta}{1-\delta} Y(\boldsymbol{e}) \geq \frac{\delta}{1-\delta} \underline{y}^B, & (15)
\end{aligned}$$

where $\overline{\beta} = \overline{\beta}(S) - \overline{\beta}(F)$, $\Delta \overline{w} = \overline{w}(S) - \overline{w}(F)$. The optimized value is the principal's payoff of the optimal relational contract.

Lemma 7 *Suppose that there is a verifiable performance measurement z_t and Assumption 2 holds. Suppose in addition that the optimal job design is task separation. Let $\boldsymbol{e} \equiv (\boldsymbol{e}_1, \boldsymbol{e}_2)$ be the* *effort and $(\beta_i(S), \beta_i(F), w_i(S), w_i(F))$ be the pair of the monetary transfer paid to agent i of the optimal relational contract. Then they solve the following optimization problem:*

$$\max_{\beta_1 \geq 0, \beta_2 \geq 0, \Delta w_1, \Delta w_2} \quad Y(\boldsymbol{e})$$

$$\textit{subject to} \quad \boldsymbol{e}_i \in \arg\max_{\tilde{\boldsymbol{e}}_i \in \mathbb{R}_+^{\#\mathcal{N}_i}} \left[\beta_i p(\tilde{\boldsymbol{e}}_i, \boldsymbol{e}_{-i}) + \Delta w_i h(\tilde{\boldsymbol{e}}_i, \boldsymbol{e}_{-i}) - \sum_{n \in \mathcal{N}_i} c_n(\tilde{e}_n) \right], \ \forall i = 1, 2, \quad (16)$$

$$-\sum_{i=1}^{2} \beta_i + \frac{\delta}{1-\delta} Y(\boldsymbol{e}) \geq \frac{\delta}{1-\delta} \underline{y}^S, \quad (17)$$

where $\beta_i = \beta_i(S) - \beta_i(F)$, $\Delta w_i = w_i(S) - w_i(F)$. The optimized value is the principal's payoff of the optimal relational contract.

Compared to cases in which there is no verifiable performance measurement, there are two additional factors in the optimization problems. First, in addition to informal bonuses, there is an explicit incentive, denoted by $\Delta \overline{w}$ or Δw_i, to optimize the principal's payoff subject to the constraints. Second, the dynamic enforcement constraint is slightly modified. After the principal reneges on an informal bonus, the parties revert to the static equilibrium outcome in which there is no informal bonus payment. Nevertheless, since there is a verifiable performance measurement, formal contracts still generate a positive amount of value, $\underline{y}^B$ or $\underline{y}^S$. Consequently, the continuation payoff on the punishment path is greater than the outside value 0. As shown in Lemma 5, the punishment payoff differs between task bundling, $\underline{y}^B$, and task separation, $\underline{y}^S$, since job design affects the multitasking incentive even through explicit incentives.

In the following, let $\boldsymbol{\Gamma}^{-1/2} \equiv (1/\sqrt{\gamma_1}, \ldots, 1/\sqrt{\gamma_N})$. Obviously, $(\boldsymbol{\Gamma}^{-1/2})^2 = \boldsymbol{\Gamma}$.

B.2.2 Task Bundling

We first characterize the optimal relational contract under task bundling by solving the optimization problem in Lemma 6. By the first order condition, the incentive compatibility constraint (14) is written by $\gamma_n e_n = \eta_n \Delta \overline{w} + \mu_n \overline{\beta}$. Plugging the incentive compatibility constraint into the objective function yields

$$\begin{aligned} \tilde{Y}^B(\overline{\beta}, \Delta \overline{w}) &\equiv \sum_n \alpha_n \frac{\mu_n \overline{\beta} + \eta_n \Delta \overline{w}}{\gamma_n} - \sum_n \frac{\gamma_n}{2} \left(\frac{\mu_n \overline{\beta} + \eta_n \Delta \overline{w}}{\gamma_n} \right)^2 \\ &= \boldsymbol{\alpha} \boldsymbol{\Gamma}^{-1} \boldsymbol{\mu}' \overline{\beta} + \boldsymbol{\alpha} \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}' \Delta \overline{w} - \frac{\boldsymbol{\mu} \boldsymbol{\Gamma}^{-1} \boldsymbol{\mu}'}{2} \overline{\beta}^2 - \frac{\boldsymbol{\eta} \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}'}{2} (\Delta \overline{w})^2 - \boldsymbol{\mu} \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}' \overline{\beta} \Delta \overline{w} \end{aligned}$$

and the dynamic enforcement constraint (15) becomes

$$\frac{1}{r}\left[\tilde{Y}^B(\overline{\beta}, \Delta\overline{w}) - \underline{y}^B\right] \geq \overline{\beta}.$$

Since $\Delta\overline{w}$ should maximize $\tilde{Y}^B(\overline{\beta}, \Delta\overline{w})$ for any $\overline{\beta}$, the first order condition with respect to $\Delta\overline{w}$ implies[18]

$$\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}' - \boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'\Delta\overline{w} - \boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'\overline{\beta} = 0 \iff \Delta\overline{w} = \frac{\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}' - \boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'\overline{\beta}}{\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'}.$$

Plugging this into $\tilde{Y}^B(\overline{\beta}, \Delta\overline{w})$ yields

$$\begin{aligned}&\frac{(\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}')^2}{2\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'} + \left(\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}' - \frac{\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'}{\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'}\right)\overline{\beta} - \frac{1}{2}\left(\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}' - \frac{(\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}')^2}{\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'}\right)\overline{\beta}^2 \\ =\ & \underline{y}^B + \hat{A}\overline{\beta} - \frac{\hat{M}}{2}\overline{\beta}^2,\end{aligned}$$

where $\hat{A} \equiv \boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}' - (\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}')/\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'$ and $\hat{M} \equiv \boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}' - (\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}')^2/\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}' \geq 0$. The dynamic enforcement constraint is similarly transformed into

$$\frac{1}{r}\left(\hat{A}\overline{\beta} - \frac{\hat{M}}{2}\overline{\beta}^2\right) \geq \overline{\beta}.$$

Note that the optimization problem has the same structure as (4) in that A is replaced with $\hat{A}$, M is replaced with $\hat{M}$, and there is a constant $\underline{y}^B$ added in the objective function. Since $\hat{A} \geq 0$ by Assumption 2, when $\hat{M} > 0$, the optimal informal bonus $\overline{\beta}$ is characterized similar to Proposition 1.[19]

When $\hat{M} = 0$, we obtain the following claim.

Claim 2 *If $\hat{M} = 0$, then $\hat{A} = 0$.*

Proof (Claim 2) *If $\hat{M} = 0$, then by the Cauchy-Schwarz inequality, two vectors $\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1/2}$ and $\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1/2}$ are linearly dependent. Then by letting $\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1/2} = \rho\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1/2}$ for some $\rho \in \mathbb{R}_{++}$, we have*

$$\hat{A} = \boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}' - \frac{\rho^2\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'}{\rho^2\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'} = 0. \square$$

Then the objective function is $\underline{y}^B$, which is obviously constant for any $\overline{\beta}$. Since $\overline{\beta} = 0$ satisfies the dynamic enforcement constraint, a contract such that $(\overline{\beta}, \Delta\overline{w}) = (0, \boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'/\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}')$ solves the optimization problem.

[18] $\tilde{Y}^B(\overline{\beta}, \Delta\overline{w})$ is concave in $(\overline{\beta}, \Delta\overline{w})$.

[19] If $\hat{A} = 0$ and $\hat{M} > 0$, then it is straightforward to see that $\overline{\beta} = 0$ is optimal.

The optimal contract is then summarized as follows.

Proposition 10 *Suppose that there is a verifiable performance measurement z_t and Assumption 2 holds. Suppose in addition that the optimal job design is task bundling. Then the optimal relational contract satisfies $\Delta\overline{w} = (\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}' - \boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'\overline{\beta})/\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'$ and*

$$(\overline{\beta}, \hat{Y}_B) = \begin{cases} \left(\frac{\hat{A}}{\hat{M}}, \frac{\hat{A}^2}{2\hat{M}} + \underline{y}^B\right) & \text{if } \hat{M} > 0 \text{ and } r < \frac{\hat{A}}{2}, \\ \left(\frac{2(\hat{A}-r)}{\hat{M}}, \frac{2r(\hat{A}-r)}{\hat{M}} + \underline{y}^B\right) & \text{if } \hat{M} > 0 \text{ and } \frac{\hat{A}}{2} \leq r < \hat{A}, \\ (0, \underline{y}^B) & \text{if } r \geq \hat{A} \text{ or } \hat{M} = 0. \end{cases}$$

$\hat{Y}^B$ is continuous in r due to the theorem of the maximum. The dynamic enforcement constraint is binding if and only if $r \geq \hat{A}/2$ or $\hat{M} = 0$.

B.2.3 Task Separation

We next characterize the optimal contract under task separation by solving the optimization problem in Lemma 7. The incentive compatibility constraint (16) is simplified by the first order condition as $\gamma_n e_n = \eta_n \Delta w_i + \mu_n \beta_i$ for $n \in \mathcal{N}_i$ and $i = 1, 2$. Plugging the incentive compatibility constraint into the objective function yields

$$\begin{aligned} &\tilde{Y}^S(\beta_1, \beta_2, \Delta w_1, \Delta w_2) \\ \equiv\ & \sum_{i=1}^{2}\left[\sum_{n\in\mathcal{N}_i}\alpha_n \frac{\mu_n\beta_i + \eta_n\Delta w_i}{\gamma_n} - \sum_{n\in\mathcal{N}_i}\frac{\gamma_n}{2}\left(\frac{\mu_n\beta_i + \eta_n\Delta w_i}{\gamma_n}\right)^2\right] \\ =\ & \sum_{i=1}^{2}\left(\boldsymbol{\alpha}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}_i'\beta_i + \boldsymbol{\alpha}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'\Delta w_i - \frac{\boldsymbol{\mu}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}_i'}{2}\beta_i^2 - \frac{\boldsymbol{\eta}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'}{2}(\Delta w_i)^2 - \boldsymbol{\mu}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'\beta_i\Delta w_i\right) \end{aligned}$$

and the dynamic enforcement constraint (17) becomes

$$\frac{1}{r}\left[\tilde{Y}^S(\beta_1, \beta_2, \Delta w_1, \Delta w_2) - \underline{y}^s\right] \geq \sum_{i=1}^{2}\beta_i.$$

Similar to task bundling, Δw_i should be chosen to maximize $\tilde{Y}^S(\beta_1, \beta_2, \Delta w_1, \Delta w_2)$. Then the first order condition with respect to Δw_i implies[20]

$$\boldsymbol{\alpha}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i' - \boldsymbol{\eta}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'\Delta w_i - \boldsymbol{\mu}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'\beta_i = 0 \iff \Delta w_i = \frac{\boldsymbol{\alpha}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i' - \boldsymbol{\mu}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'\beta_i}{\boldsymbol{\eta}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'}.$$

[20] $\tilde{Y}^S(\beta_1, \beta_2, \Delta w_1, \Delta w_2)$ is concave in $(\beta_1, \beta_2, \Delta w_1, \Delta w_2)$.

Plugging this into $\tilde{Y}^S(\beta_1, \beta_2, \Delta w_1, \Delta w_2)$ yields

$$\sum_{i=1}^{2} \left[\frac{(\boldsymbol{\alpha}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i')^2}{2\boldsymbol{\eta}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i'} + \left(\boldsymbol{\alpha}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\mu}_i' - \frac{\boldsymbol{\alpha}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i' \boldsymbol{\mu}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i'}{\boldsymbol{\eta}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i'} \right) \beta_i - \frac{1}{2} \left(\boldsymbol{\mu}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\mu}_i' - \frac{(\boldsymbol{\mu}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i')^2}{\boldsymbol{\eta}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i'} \right) \beta_i^2 \right]$$

$$= \underline{y}^S + \sum_{i=1}^{2} \left(\hat{a}_i \beta_i - \frac{\hat{m}_i}{2} \beta_i^2 \right),$$

where $\hat{a}_i \equiv \boldsymbol{\alpha}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\mu}_i' - (\boldsymbol{\alpha}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i' \boldsymbol{\mu}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i') / \boldsymbol{\eta}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i'$ and $\hat{m}_i \equiv \boldsymbol{\mu}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\mu}_i' - (\boldsymbol{\mu}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i')^2 / \boldsymbol{\eta}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i' \geq 0$. The dynamic enforcement constraint is similarly transformed into

$$\frac{1}{r} \sum_{i=1}^{2} \left(\hat{a}_i \beta_i - \frac{\hat{m}_i}{2} \beta_i^2 \right) \geq \sum_{i=1}^{2} \beta_i.$$

Similar to task bundling, the optimization problem has the same structure as (8) in that a_i is replaced with $\hat{a}_i$, m_i is replaced with $\hat{m}_i$, and there is a constant $\underline{y}^S$ added in the objective function. Since $\hat{a}_i \geq 0$ by Assumption 2, when $\hat{m}_i > 0$ for both $i = 1, 2$, the optimal informal bonus (β_1, β_2) is characterized similar to Proposition 2.[21]

If $\hat{m}_j = 0$ for some $j = 1, 2$, then by the same procedure as the proof of Claim 2, we obtain the following claim.

Claim 3 *If $\hat{m}_j = 0$, then $\hat{a}_j = 0$.*

Then the optimization problem is transformed into:

$$\max_{\beta_1, \beta_2} \underline{y}^S + \hat{a}_i \beta_i - \frac{\hat{m}_i}{2} \beta_i^2 \text{ subject to } \frac{1}{r} \left(\hat{a}_i \beta_i - \frac{\hat{m}_i}{2} \beta_i^2 \right) \geq \sum_{\ell=1}^{2} \beta_\ell.$$

Since β_j appears only in the right hand side of the constraint, $\beta_j = 0$. Given $\beta_j = 0$, note that now the optimization problem has the same structure as task bundling in that $\hat{A}$ is replaced with $\hat{a}_i$, $\hat{M}$ is replaced with $\hat{m}_i$, and $\underline{y}^B$ is replaced with $\underline{y}^S$. The optimal informal bonus (β_1, β_2) is then characterized similar to Proposition 10.

Therefore the optimal contract under task separation is summarized as follows.

Proposition 11 *Suppose that there is a verifiable performance measurement z_t, Assumption 2 holds, and $\hat{a}_1 \geq \hat{a}_2$. Suppose in addition that the optimal job design is task separation. Then the optimal relational contract satisfies $\Delta w_i = (\boldsymbol{\alpha}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i' - \boldsymbol{\mu}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i' \beta) / \boldsymbol{\eta}_i \boldsymbol{\Gamma}^{-1} \boldsymbol{\eta}_i'$ for $i = 1, 2$ and the following.*

[21] If $\hat{a}_i > 0$ for some i, then the proof of Proposition 2 still applies. If $\hat{a}_1 = \hat{a}_2 = 0$, then it is straightforward to see that $\beta_i = 0$ for $i = 1, 2$ is optimal.

- *When $\hat{m}_i > 0$ for both $i = 1,2$,*

$$(\beta_1, \beta_2) = \begin{cases} \left(\dfrac{\hat{a}_1}{\hat{m}_1}, \dfrac{\hat{a}_2}{\hat{m}_2}\right) & \text{if } \hat{a}_i \neq 0 \text{ for some } i = 1,2 \\ & \text{and } r < \dfrac{\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1}{2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1)}, \\ \left(\dfrac{\hat{a}_1 - r + \hat{K}(r)}{\hat{m}_1}, \dfrac{\hat{a}_2 - r + \hat{K}(r)}{\hat{m}_2}\right) & \text{if } \hat{a}_i \neq 0 \text{ for some } i = 1,2 \\ & \text{and } \dfrac{(\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1)}{2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1)} \leq r < \dfrac{\sum_i \hat{a}_i}{2}, \\ \left(\dfrac{2(\hat{a}_1 - r)}{\hat{m}_1}, 0\right) & \text{if } \dfrac{\sum_i \hat{a}_i}{2} \leq r < \hat{a}_1, \\ (0,0) & \text{if } r \geq \hat{a}_1, \end{cases}$$

where $\hat{K}(r) \equiv \sqrt{[(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1)/(\sum_i \hat{m}_i) - r]^2 + [\hat{m}_1\hat{m}_2(\hat{a}_1 - \hat{a}_2)^2]/(\sum_i \hat{m}_i)^2}$ and the principal's payoff satisfies

$$\hat{Y}^S = \begin{cases} \dfrac{\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1}{2\hat{m}_1\hat{m}_2} + \underline{y}^S & \text{if } \hat{a}_i \neq 0 \text{ for some } i = 1,2 \\ & \text{and } r < \dfrac{\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1}{2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1)}, \\ \dfrac{r\left[\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1 - \sum_i \hat{m}_i[r - \hat{K}(r)]\right]}{\hat{m}_1\hat{m}_2} + \underline{y}^S & \text{if } \hat{a}_i \neq 0 \text{ for some } i = 1,2 \\ & \text{and } \dfrac{\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1}{2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1)} \leq r < \dfrac{\sum_i \hat{a}_i}{2}, \\ \dfrac{2r(\hat{a}_1 - r)}{\hat{m}_1} + \underline{y}^S & \text{if } \dfrac{\sum_i \hat{a}_i}{2} \leq r \leq \hat{a}_1, \\ \underline{y}^S & \text{if } r > \hat{a}_1. \end{cases}$$

- *When $\hat{m}_j = 0$ for some $j = 1,2$, $\beta_j = 0$ and*

$$(\beta_i, \hat{Y}_S) = \begin{cases} \left(\dfrac{\hat{a}_i}{\hat{m}_i}, \dfrac{\hat{a}_i^2}{2\hat{m}_i} + \underline{y}^S\right) & \text{if } \hat{m}_i > 0 \text{ and } r < \dfrac{\hat{a}_i}{2}, \\ \left(\dfrac{2(\hat{a}_i - r)}{\hat{m}_i}, \dfrac{2r(\hat{a}_i - r)}{\hat{m}_i} + \underline{y}^S\right) & \text{if } \hat{m}_i > 0 \text{ and } \dfrac{\hat{a}_i}{2} \leq r < \hat{a}_i, \\ (0, \underline{y}^S) & \text{if } r \geq \hat{a}_i \text{ or } \hat{m}_i = 0. \end{cases}$$

$\hat{Y}^S$ is continuous in r due to the theorem of the maximum. The dynamic enforcement constraint is binding if and only if $r \geq (\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1)/[2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1)]$ or $\hat{m}_1 = \hat{m}_2 = 0$.

B.3 Proof of Proposition 7

First suppose $\hat{M} = 0$, which is equivalent to that $\boldsymbol{\mu}$ and $\boldsymbol{\eta}$ are linearly dependent. Then, by construction, $\boldsymbol{\mu}_i$ and $\boldsymbol{\eta}_i$ are also linearly dependent for $i = 1,2$, which implies

$\hat{m}_1 = \hat{m}_2 = 0$. Then from Propositions 10 and 11, for all r, $\hat{Y}^S = \underline{y}^S \geq \underline{y}^B = \hat{Y}^B$.

Next suppose $\hat{M} > 0$. Let

$$\hat{A}^E \equiv \boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}' - \frac{\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'}{\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'}, \qquad \hat{M}^E \equiv \boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}' - \frac{(\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}')^2}{\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'},$$

where $\hat{M}^E > 0$ since $\hat{M} > 0$. Note that for any $r > 0$,

$$\begin{aligned}
&\max_{\beta_1,\beta_2,\Delta w_1,\Delta w_2} \tilde{Y}^S(\beta_1,\beta_2,\Delta w_1,\Delta w_2) \geq \tilde{Y}^S\left(\frac{\hat{A}}{\hat{M}},\frac{\hat{A}}{\hat{M}},\frac{\hat{A}^E}{\hat{M}^E},\frac{\hat{A}^E}{\hat{M}^E}\right) \\
=\;& \boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'\frac{\hat{A}}{\hat{M}} + \boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'\frac{\hat{A}^E}{\hat{M}^E} - \frac{\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'}{2}\left(\frac{\hat{A}}{\hat{M}}\right)^2 - \frac{\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'}{2}\left(\frac{\hat{A}^E}{\hat{M}^E}\right)^2 - \boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'\frac{\hat{A}}{\hat{M}}\frac{\hat{A}^E}{\hat{M}^E} \\
=\;& \tilde{Y}^B\left(\frac{\hat{A}}{\hat{M}},\frac{\hat{A}^E}{\hat{M}^E}\right) \geq \hat{Y}^B.
\end{aligned}$$

We now show $\hat{Y}^S = \max_{\beta_1,\beta_2,\Delta w_1,\Delta w_2} \tilde{Y}^S(\beta_1,\beta_2,\Delta w_1,\Delta w_2)$ for sufficiently small r, by which we show $\hat{Y}^S \geq \hat{Y}^B$ for sufficiently small r. If $\hat{m}_1 = \hat{m}_2 = 0$, then $\boldsymbol{\mu}_i\boldsymbol{\Gamma}^{-1/2}$ and $\boldsymbol{\eta}_i\boldsymbol{\Gamma}^{-1/2}$ are linearly dependent for both $i = 1,2$, which implies that $\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1/2}$ and $\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1/2}$ are also linearly dependent and then $\hat{M} = 0$, a contradiction. Then $\hat{m}_i > 0$ at least for some i. When $\hat{m}_i > 0$ for both $i = 1,2$, from Proposition 11, we see that $\hat{Y}^S = \max_{\beta_1,\beta_2,\Delta w_1,\Delta w_2} \tilde{Y}^S(\beta_1,\beta_2,\Delta w_1,\Delta w_2)$ when (i) $\hat{a}_i \neq 0$ for some $i = 1,2$, and $r \leq (\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1)/[2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1)]$; or (ii) $\hat{a}_i = 0$ for both $i = 1,2$ (no matter what r is). When $\hat{m}_i > \hat{m}_j = 0$, by Proposition 11, $\hat{Y}^S = \max_{\beta_1,\beta_2,\Delta w_1,\Delta w_2} \tilde{Y}^S(\beta_1,\beta_2,\Delta w_1,\Delta w_2)$ when (i) $\hat{a}_i \neq 0$ and $r \leq \hat{a}_i/2$; or (ii) $\hat{a}_i = 0$ (no matter what r is). Therefore in any cases, $\hat{Y}^S = \max_{\beta_1,\beta_2,\Delta w_1,\Delta w_2} \tilde{Y}^S(\beta_1,\beta_2,\Delta w_1,\Delta w_2)$ when r is sufficiently small.

To show $\hat{Y}^S \geq \hat{Y}^B$ for sufficiently large r, let $r \geq \max\{\hat{A}, \hat{a}_i\}$. Then we have $\hat{Y}^S = \underline{y}^S \geq \underline{y}^B = \hat{Y}^B$ by Lemma 5.

B.4 Proof of Proposition 8

First, the quasi-symmetry implies the following property.

Lemma 8 *Under Assumption 3,* $\hat{A} = \sum_{i=1}^2 \hat{a}_i$ *and* $\hat{M} = \sum_{i=1}^2 \hat{m}_i$.

Proof (Lemma 8) *Since* $\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}' = \sum_{i=1}^2 \alpha_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}_i'$*, and* $\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'$*,* $\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'$*, and* $\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'$ *can be similarly expressed by the additively separable way,*

$$\sum_i \hat{a}_i - \hat{A} = \frac{\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'}{\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'} - \sum_i \frac{\alpha_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'\boldsymbol{\mu}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'}{\boldsymbol{\eta}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'}$$

$$= \frac{\left(\sum_i \boldsymbol{\alpha}_i \boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'\right)\left(\sum_i \boldsymbol{\mu}_i \boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'\right)}{\sum_i \boldsymbol{\eta}_i \boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'} - \sum_i \frac{\boldsymbol{\alpha}_i \boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i' \boldsymbol{\mu}_i \boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'}{\boldsymbol{\eta}_i \boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'}$$

$$= \frac{\boldsymbol{\alpha}_1 \boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_1'\boldsymbol{\eta}_2\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_2' - \boldsymbol{\alpha}_2 \boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_2'\boldsymbol{\eta}_1\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_1'}{\sum_i \boldsymbol{\eta}_i \boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'} \left(\frac{\boldsymbol{\mu}_1\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_1'}{\boldsymbol{\eta}_1\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_1'} - \frac{\boldsymbol{\mu}_2\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_2'}{\boldsymbol{\eta}_2\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_2'}\right) = 0,$$

where the last equality is due to Assumption 3. Similarly,

$$\sum_i \hat{m}_i - \hat{M} = \frac{\left(\boldsymbol{\mu}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'\right)^2}{\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'} - \sum_i \frac{\left(\boldsymbol{\mu}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'\right)^2}{\boldsymbol{\eta}_i\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'},$$

which is equivalent to $\sum_i \hat{a}_i - \hat{A}$ where $\boldsymbol{\alpha}$ is replaced with $\boldsymbol{\mu}$ and $\boldsymbol{\alpha}_i$ is replaced with $\boldsymbol{\mu}_i$, implying

$$\frac{\boldsymbol{\mu}_1 \boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_1'\boldsymbol{\eta}_2\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_2' - \boldsymbol{\mu}_2 \boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_2'\boldsymbol{\eta}_1\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_1'}{\sum_i \boldsymbol{\eta}_i \boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_i'} \left(\frac{\boldsymbol{\mu}_1\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_1'}{\boldsymbol{\eta}_1\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_1'} - \frac{\boldsymbol{\mu}_2\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_2'}{\boldsymbol{\eta}_2\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}_2'}\right) = 0. \square$$

In the following, we compare $\hat{Y}^B$ and $\hat{Y}^S$ characterized by Propositions 10 and 11, where without loss of generality we assume $\hat{a}_1 \geq \hat{a}_2$ in Proposition 11. Note that Lemma 5 implies $\underline{y}^S > \underline{y}^B$ since (12) is satisfied. There are several cases to be considered.

B.4.1 $\hat{M} = 0$ or $\hat{a}_1 = \hat{a}_2 = 0$

In this case, for any r, $\hat{Y}^S = \underline{y}^S > \underline{y}^B = \hat{Y}^B$.

In the rest of cases, we assume $\hat{M} > 0$ and $\hat{a}_1 > 0$.

B.4.2 $\underline{y}^S - \underline{y}^B > \hat{A}^2/2\hat{M}$

Note that $\hat{Y}^S$ and $\hat{Y}^B$ are non-increasing in r. Since for any r, $\hat{Y}^S \geq \underline{y}^S$ and $\hat{Y}^B \leq \hat{A}^2/2\hat{M} + \underline{y}^B$, $\hat{Y}^S \geq \underline{y}^S > \hat{A}^2/2\hat{M} + \underline{y}^B \geq \hat{Y}^B$.

In the rest of cases, we assume $\underline{y}^S \leq \hat{A}^2/2\hat{M} + \underline{y}^B$.

B.4.3 $\hat{m}_i > 0$ for $i = 1, 2$

We first show a series of claims.

Claim 4 *Under Assumptions 2 and 3, when $\hat{m}_i > 0$ for $i = 1, 2$ and $\hat{a}_1 > 0$, $\hat{Y}^S > \hat{Y}^B$ for $r \leq (\hat{m}_1\hat{a}_2^2 + \hat{m}_2\hat{a}_1^2)/[2(\hat{m}_1\hat{a}_2 + \hat{m}_2\hat{a}_1)]$.*

Proof (Claim 4) *Note that Lemma 8 implies $(\hat{m}_1\hat{a}_2^2 + \hat{m}_2\hat{a}_1^2)/(\hat{m}_1\hat{a}_2 + \hat{m}_2\hat{a}_1) < \hat{A}$. Then, for $r \leq (\hat{m}_1\hat{a}_2^2 + \hat{m}_2\hat{a}_1^2)/[2(\hat{m}_1\hat{a}_2 + \hat{m}_2\hat{a}_1)]$, by Propositions 10 and 11 and Lemmas 5 and 8,*

$$\hat{Y}^S - \hat{Y}^B = \frac{1}{2}\left(\sum_{i=1}^{2} \frac{\hat{a}_i^2}{\hat{m}_i} - \frac{\hat{A}^2}{\hat{M}}\right) + \underline{y}^S - \underline{y}^B = \frac{(\hat{a}_1\hat{m}_2 - \hat{a}_2\hat{m}_1)^2}{2\hat{m}_1\hat{m}_2\hat{M}} + \underline{y}^S - \underline{y}^B > 0. \square$$

Claim 5 *Under Assumptions 2 and 3, when* $\hat{m}_i > 0$ *for* $i = 1, 2$, $\underline{y}^S \leq \hat{A}^2/2\hat{M} + \underline{y}^B$, $\hat{a}_1 \geq \hat{a}_2$, *and* $\hat{a}_1 > 0$,

1. $\hat{Y}^S - \hat{Y}^B$ *is decreasing in* $r \in ((\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1)/2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1), \hat{A}/2]$; *and*
2. $\hat{Y}^S \leq \hat{Y}^B$ *at* $r = \hat{A}/2$ *if and only if*

$$\underline{y}^S - \underline{y}^B \leq \frac{\hat{A}}{\hat{M}}\left[\hat{a}_2 - \frac{\hat{m}_2}{2\hat{m}_1}(\hat{a}_1 - \hat{a}_2)\right]. \tag{18}$$

Proof (Claim 5) *From Propositions 10 and 11 and Lemma 8, for* $r \in ((\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1)/2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1), \hat{A}/2]$, $\hat{Y}^S$ *is decreasing in r while* $\hat{Y}^B$ *is constant, which implies that* $\hat{Y}^S - \hat{Y}^B$ *is decreasing in r. Furthermore,* $\hat{Y}^S \leq \hat{Y}^B$ *at* $r = \hat{A}/2$ *if and only if*

$$\frac{\hat{A}\left[(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1) - \sum_i \hat{m}_i[\hat{A}/2 - \hat{K}(\hat{A}/2)]\right]}{2\hat{m}_1\hat{m}_2} + \underline{y}^S \leq \frac{\hat{A}^2}{2\hat{M}} + \underline{y}^B$$
$$\iff \quad \hat{K}\left(\frac{\hat{A}}{2}\right) \leq \frac{\hat{A}\hat{m}_1\hat{m}_2}{\hat{M}^2} + \frac{\hat{A}}{2} - \frac{\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1}{\hat{M}} - \frac{2\hat{m}_1\hat{m}_2}{\hat{A}\hat{M}}(\underline{y}^S - \underline{y}^B), \tag{19}$$

which is satisfied only if the right hand side is non-negative. Given that the right hand side is non-negative, this is satisfied if and only if

$$\left(\frac{\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1}{\hat{M}} - \frac{\hat{A}}{2}\right)^2 + \frac{\hat{m}_1\hat{m}_2(\hat{a}_1 - \hat{a}_2)^2}{\hat{M}^2}$$
$$\leq \left[\frac{\hat{A}\hat{m}_1\hat{m}_2}{\hat{M}^2} + \frac{\hat{A}}{2} - \frac{\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1}{M} - \frac{2\hat{m}_1\hat{m}_2}{\hat{A}\hat{M}}(\underline{y}^S - \underline{y}^B)\right]^2$$
$$\iff \quad \left[\frac{\hat{A}\hat{m}_1\hat{m}_2}{\hat{M}^2} - \frac{2\hat{m}_1\hat{m}_2}{\hat{A}\hat{M}}(\underline{y}^S - \underline{y}^B)\right]^2 + 2\left[\frac{\hat{A}\hat{m}_1\hat{m}_2}{\hat{M}^2} - \frac{2\hat{m}_1\hat{m}_2}{\hat{A}\hat{M}}(\underline{y}^S - \underline{y}^B)\right]\left(\frac{\hat{A}}{2} - \frac{\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1}{\hat{M}}\right)$$
$$\geq \frac{\hat{m}_1\hat{m}_2(\hat{a}_1 - \hat{a}_2)^2}{\hat{M}^2}$$
$$\iff \quad \frac{4\hat{m}_1^2\hat{m}_2^2}{\hat{A}^2\hat{M}^2}\left[\frac{\hat{A}^2}{2\hat{M}} - (\underline{y}^S - \underline{y}^B) + \frac{\hat{A}(\hat{a}_1 - \hat{a}_2)}{2\hat{m}_2}\right]\left[\frac{\hat{A}^2}{2\hat{M}} - (\underline{y}^S - \underline{y}^B) - \frac{\hat{A}(\hat{a}_1 - \hat{a}_2)}{2\hat{m}_1}\right] \geq 0.$$

Note that the first term in square brackets is non-negative since $\hat{a}_1 \geq \hat{a}_2$ *and* $\underline{y}^S \leq \hat{A}^2/2\hat{M} + \underline{y}^B$. *Then the inequality is satisfied if and only if*

$$\frac{\hat{A}^2}{2\hat{M}} - (\underline{y}^S - \underline{y}^B) - \frac{\hat{A}(\hat{a}_1 - \hat{a}_2)}{2\hat{m}_1} \geq 0,$$

which is equivalent to (18). Finally, given (18), the right hand side of (19) is positive since

$$\frac{\hat{A}\hat{m}_1\hat{m}_2}{\hat{M}^2} + \frac{\hat{A}}{2} - \frac{\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1}{\hat{M}} - \frac{2\hat{m}_1\hat{m}_2}{\hat{A}\hat{M}}(\underline{y}^S - \underline{y}^B)$$

$$\geq \quad \frac{\hat{A}\hat{m}_1\hat{m}_2}{\hat{M}^2} + \frac{\hat{A}}{2} - \frac{\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1}{\hat{M}} - \frac{2\hat{m}_1\hat{m}_2}{\hat{A}\hat{M}}\left\{\frac{\hat{A}}{\hat{M}}\left[\hat{a}_2 - \frac{\hat{m}_2}{2\hat{m}_1}(\hat{a}_1 - \hat{a}_2)\right]\right\}$$
$$= \quad \frac{\hat{a}_1 - \hat{a}_2}{2} \geq 0. \ \square$$

Claim 6 *Under Assumptions 2 and 3, when $\hat{m}_i > 0$ for $i = 1, 2$,*

1. *$\hat{Y}^S - \hat{Y}^B$ is decreasing in $r \in (\hat{A}/2, \hat{a}_1]$; and*

2. *$\hat{Y}^S \gtreqless \hat{Y}^B$ at $r = \hat{a}_1$ if and only if $\underline{y}^S - \underline{y}^B \gtreqless 2\hat{a}_1\hat{a}_2/\hat{M}$.*

Proof (Claim 6) *From Propositions 10 and 11 and Lemma 8, for $r \in (\hat{A}/2, \hat{a}_1]$,*

$$\hat{Y}^S - \hat{Y}^B = \frac{2r(\hat{a}_1 - r)}{\hat{m}_1} - \frac{2r(\hat{A} - r)}{\hat{M}} + \underline{y}^S - \underline{y}^B = -\frac{2\hat{m}_2}{\hat{m}_1\hat{M}}r^2 + \frac{2(\hat{a}_1\hat{m}_2 - \hat{a}_2\hat{m}_1)}{\hat{m}_1\hat{M}}r + \underline{y}^S - \underline{y}^B.$$

This is decreasing in $r \in (\hat{A}/2, \hat{a}_1]$ since

$$\frac{\partial}{\partial r}(\hat{Y}^S - \hat{Y}^B) = -\frac{4\hat{m}_2}{\hat{m}_1\hat{M}}r + \frac{2(\hat{a}_1\hat{m}_2 - \hat{a}_2\hat{m}_1)}{\hat{m}_1\hat{M}} < -\frac{4\hat{m}_2}{\hat{m}_1\hat{M}}\frac{\hat{A}}{2} + \frac{2(\hat{a}_1\hat{m}_2 - \hat{a}_2\hat{m}_1)}{\hat{m}_1\hat{M}} = -\frac{2\hat{a}_2}{\hat{m}_1} < 0.$$

Furthermore, at $r = \hat{a}_1$,

$$\hat{Y}^S - \hat{Y}^B = -\frac{2\hat{a}_1\hat{a}_2}{\hat{M}} + \underline{y}^S - \underline{y}^B,$$

which obviously implies that $\hat{Y}^S \gtreqless \hat{Y}^B$ if and only if $\underline{y}^S - \underline{y}^B \gtreqless 2\hat{a}_1\hat{a}_2/\hat{M}$. $\square$

Claim 7 *Under Assumptions 2 and 3, when $\hat{m}_i > 0$ for $i = 1, 2$,*

1. *$\hat{Y}^S - \hat{Y}^B$ is increasing in $r \in (\hat{a}_1, \hat{A}]$; and*

2. *$\hat{Y}^S > \hat{Y}^B$ at $r = \hat{A}$.*

Proof (Claim 7) *From Propositions 10 and 11 and Lemma 8, for $r \in (\hat{a}_1, \hat{A}]$, $\hat{Y}^B$ is decreasing in r while $\hat{Y}^S$ is constant, which implies that $\hat{Y}^S - \hat{Y}^B$ is increasing in r. Furthermore, at $r = \hat{A}$, $\hat{Y}^S = \underline{y}^S > \underline{y}^B = \hat{Y}^B$.* $\square$

Now we provide the comparison of $\hat{Y}^B$ and $\hat{Y}^S$. By Claim 4, $\hat{Y}^S > \hat{Y}^B$ for $r \leq ((\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1)/2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1)$. Moreover, for $r > \hat{A}$, $\hat{Y}^S = \underline{y}^S > \underline{y}^B = \hat{Y}^B$. The rest of the cases depends on $\underline{y}^S - \underline{y}^B$. Note that given $\hat{a}_1 \geq \hat{a}_2$,

$$\frac{2\hat{a}_1\hat{a}_2}{\hat{M}} - \frac{\hat{A}}{\hat{M}}\left[\hat{a}_2 - \frac{\hat{m}_2}{2\hat{m}_1}(\hat{a}_1 - \hat{a}_2)\right] = \frac{1}{\hat{M}}\left(\hat{a}_2 + \frac{\hat{A}\hat{m}_2}{2\hat{m}_1}\right)(\hat{a}_1 - \hat{a}_2) \geq 0.$$

1. Suppose

$$\underline{y}^S - \underline{y}^B \leq \frac{\hat{A}}{\hat{M}}\left[\hat{a}_2 - \frac{\hat{m}_2}{2\hat{m}_1}(\hat{a}_1 - \hat{a}_2)\right].$$

   (a) For $r \in ((\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1)/2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1), \hat{A}/2]$, Claim 5 implies that $\hat{Y}^S - \hat{Y}^B$ is decreasing in r and $\hat{Y}^S \leq \hat{Y}^B$ at $r = \hat{A}/2$. Since $\hat{Y}^S > \hat{Y}^B$ at $r = (\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1)/2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1)$, there uniquely exists $\tilde{r}_1 \in ((\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1)/2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1, \hat{A}/2]$ such that $\hat{Y}^S \gtreqless \hat{Y}^B$ if and only if $r \lesseqgtr \tilde{r}_1$.

   (b) For $r \in (\hat{A}/2, \hat{a}_1]$, Claim 6 implies that $\hat{Y}^S - \hat{Y}^B$ is decreasing in r. Since $\hat{Y}^S \leq \hat{Y}^B$ at $r = \hat{A}/2$, $\hat{Y}^S < \hat{Y}^B$ for all $r \in (\hat{A}/2, \hat{a}_1]$.

   (c) For $r \in (\hat{a}_1, \hat{A}]$, Claim 7 implies that $\hat{Y}^S - \hat{Y}^B$ is increasing in r and $\hat{Y}^S > \hat{Y}^B$ at $r = \hat{A}$. Since $\hat{Y}^S < \hat{Y}^B$ at $r = \hat{a}_1$, there uniquely exists $\tilde{r}_3 \in (\hat{a}_1, \hat{A})$ such that $\hat{Y}^S \gtreqless \hat{Y}^B$ if and only if $r \gtreqless \tilde{r}_3$.

2. Suppose

$$\frac{\hat{A}}{\hat{M}}\left[\hat{a}_2 - \frac{\hat{m}_2}{2\hat{m}_1}(\hat{a}_1 - \hat{a}_2)\right] < \underline{y}^S - \underline{y}^B \leq \frac{2\hat{a}_1\hat{a}_2}{\hat{M}}.$$

   (a) For $r \in ((\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1)/2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1), \hat{A}/2]$, Claim 5 implies that $\hat{Y}^S - \hat{Y}^B$ is decreasing in r and $\hat{Y}^S > \hat{Y}^B$ at $r = \hat{A}/2$ since (18) is violated. Then $\hat{Y}^S > \hat{Y}^B$ for all $r \in ((\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1)/2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1), \hat{A}/2]$.

   (b) For $r \in (\hat{A}/2, \hat{a}_1]$, Claim 6 implies that $\hat{Y}^S - \hat{Y}^B$ is decreasing in r and $\hat{Y}^S \leq \hat{Y}^B$ at $r = \hat{a}_1$. Since $\hat{Y}^S > \hat{Y}^B$ at $r = \hat{A}/2$, there uniquely exists $\tilde{r}_2 \in (\hat{A}/2, \hat{a}_1]$ such that $\hat{Y}^S \gtreqless \hat{Y}^B$ if and only if $r \lesseqgtr \tilde{r}_2$.

   (c) For $r \in (\hat{a}_1, \hat{A}]$, Claim 7 implies that $\hat{Y}^S - \hat{Y}^B$ is increasing in r and $\hat{Y}^S > \hat{Y}^B$ at $r = \hat{A}$. Since $\hat{Y}^S \leq \hat{Y}^B$ at $r = \hat{a}_1$, there uniquely exists $\tilde{r}_3 \in (\hat{a}_1, \hat{A})$ such that $\hat{Y}^S \gtreqless \hat{Y}^B$ if and only if $r \gtreqless \tilde{r}_3$.

3. Suppose

$$\underline{y}^S - \underline{y}^B > \frac{2\hat{a}_1\hat{a}_2}{\hat{M}}.$$

   (a) For $r \in ((\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1)/2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1), \hat{A}/2]$, as in the last case, $\hat{Y}^S > \hat{Y}^B$ for all $r \in ((\hat{a}_1^2\hat{m}_2 + \hat{a}_2^2\hat{m}_1)/2(\hat{a}_1\hat{m}_2 + \hat{a}_2\hat{m}_1), \hat{A}/2]$.

   (b) For $r \in (\hat{A}/2, \hat{a}_1]$, Claim 6 implies that $\hat{Y}^S - \hat{Y}^B$ is decreasing in r and $\hat{Y}^S > \hat{Y}^B$ at $r = \hat{a}_1$. Then, $\hat{Y}^S > \hat{Y}^B$ for all $r \in (\hat{A}/2, \hat{a}_1]$.

(c) For $r \in (\hat{a}_1, \hat{A}]$, Claim 7 implies that $\hat{Y}^S - \hat{Y}^B$ is increasing in r. Since $\hat{Y}^S > \hat{Y}^B$ at $r = \hat{a}_1$, $\hat{Y}^S > \hat{Y}^B$ for all $r \in (\hat{a}_1, \hat{A}]$.

The summary of the results are as follows.[22]

1. If $\underline{y}^S - \underline{y}^B \leq 2\hat{a}_1\hat{a}_2/\hat{M}$, then there exist $\underline{r} > 0$ and $\overline{r} \geq \underline{r}$ such that $\hat{Y}^S < \hat{Y}^B$ for $r \in (\underline{r}, \overline{r})$, $\hat{Y}^S = \hat{Y}^B$ for $r = \underline{r}, \overline{r}$, and $\hat{Y}^S > \hat{Y}^B$ otherwise.

2. Otherwise, $\hat{Y}^S > \hat{Y}^B$ for all r.

B.4.4 $\hat{m}_\ell = 0$ for some $\ell = 1, 2$

Let $k = 1, 2$ such that $k \neq \ell$. Claim 3 implies $\hat{a}_\ell = 0$. Then Lemma 8 implies $\hat{m}_k = \hat{M} > 0$ and $\hat{a}_k = \hat{A} > 0$. Hence from Proposition 11,

$$\hat{Y}^S = \begin{cases} \dfrac{\hat{A}^2}{2\hat{M}} + \underline{y}^S & \text{if } r \leq \dfrac{\hat{A}}{2}, \\ \dfrac{2r(\hat{A} - r)}{\hat{M}} + \underline{y}^S & \text{if } \dfrac{\hat{A}}{2} < r \leq \hat{A}, \\ \underline{y}^S & \text{if } r \geq \hat{A}. \end{cases}$$

Therefore for any r, $\hat{Y}^S - \hat{Y}^B = \underline{y}^S - \underline{y}^B > 0$.

B.4.5 Characterization of $\hat{y}$

Recall that $\hat{M} > 0$ if $\boldsymbol{\mu}$ and $\boldsymbol{\eta}$ are linearly independent. Given $\hat{M} > 0$, let $\hat{y} \equiv 2\hat{a}_1\hat{a}_2/\hat{M}$, by which the proof is completed.[23]

B.5 Proof of Proposition 9

Under Assumption 4, there is $\rho \in \mathbb{R}_{++}$ such that $\boldsymbol{\mu} = \rho\boldsymbol{\alpha}$ and $\boldsymbol{\mu}_i = \rho\boldsymbol{\alpha}_i$ for $i = 1, 2$. By plugging them into $\hat{M}$ and $\hat{m}_i$, we obtain $\hat{M} = \rho\hat{A}$ and $\hat{m}_i = \rho\hat{a}_i$ for $i = 1, 2$. Note that since $\boldsymbol{\alpha}$ and $\boldsymbol{\eta}$ are linearly independent, the Cauchy-Schwarz inequality implies

[22]Solving the equations yields the explicit expression of $\tilde{r}_1$, $\tilde{r}_2$, and $\tilde{r}_3$ above as

$$\tilde{r}_1 \equiv \left[\frac{\hat{A}^2}{2\hat{M}} - (\underline{y}^S - \underline{y}^B)\right]\left[\frac{\hat{a}_1}{\hat{m}_1} + \frac{\hat{a}_2}{\hat{m}_2} + \sqrt{\left(\frac{\hat{a}_1}{\hat{m}_1} - \frac{\hat{a}_2}{\hat{m}_2}\right)^2 + \frac{2\hat{M}(\underline{y}^S - \underline{y}^B)}{\hat{m}_1\hat{m}_2}}\right]^{-1},$$

$$\tilde{r}_2 \equiv \frac{1}{2}\left[\hat{a}_1 - \frac{\hat{a}_2\hat{m}_1}{\hat{m}_2} + \sqrt{\left(\frac{\hat{a}_1\hat{m}_2 - \hat{a}_2\hat{m}_1}{\hat{m}_2}\right)^2 + \frac{2\hat{m}_1\hat{M}(\underline{y}^S - \underline{y}^B)}{\hat{m}_2}}\right], \tilde{r}_3 \equiv \frac{\hat{A} + \sqrt{\hat{A}^2 - 2(\underline{y}^S - \underline{y}^B)\hat{M}}}{2}.$$

[23]Note that if $\hat{m}_\ell = 0$ then $\hat{y} = 0$ since Claim 3 implies $\hat{a}_\ell = 0$. Hence $\underline{y}^S - \underline{y}^B > \hat{y}$ given that (12) is satisfied.

$\hat{A} > 0$.[24] Note also that Assumption 4 implies (10) and (11). Then from Proposition 10, $\hat{Y}^B$ is written as

$$\hat{Y}^B = \begin{cases} \dfrac{\hat{A}}{2\rho} + \underline{y}^B & \text{if } r < \dfrac{\hat{A}}{2}, \\ \dfrac{2r(\hat{A} - r)}{\rho \hat{A}} + \underline{y}^B & \text{if and } \dfrac{\hat{A}}{2} \leq r < \hat{A}, \\ \underline{y}^B & \text{if } r \geq \hat{A}. \end{cases}$$

Note also that $\hat{A} - \sum_i \hat{a}_i = 2\rho(\underline{y}^S - \underline{y}^B) \geq 0$ under Assumption 4.

Recall that in Proposition 11, without loss of generality we assume $\hat{a}_1 \geq \hat{a}_2$, which we maintain in the following. Note that by the Cauchy-Schwarz inequality, $\boldsymbol{\alpha}_i$ and $\boldsymbol{\eta}_i$ are linearly dependent if and only if $\hat{a}_i = 0$. If $\boldsymbol{\alpha}^i$ and $\boldsymbol{\eta}^i$ are also linearly dependent for both $i = 1, 2$, then so are $\boldsymbol{\alpha}$ and $\boldsymbol{\eta}$, which contradicts the assumption. Since $\hat{a}_1 \geq \hat{a}_2$ and $\hat{a}_\ell \geq 0$ for both $\ell = 1, 2$, $\boldsymbol{\alpha}_i$ and $\boldsymbol{\eta}_i$ are linearly dependent only if $i = 2$.

There are two cases to be considered: cases of $\hat{a}_2 > 0$ and $\hat{a}_2 = 0$.

B.5.1 $\hat{a}_2 > 0$

Since $\hat{a}_1 \geq \hat{a}_2 > 0$, by Proposition 11, $\hat{Y}^S$ is written as

$$\hat{Y}^S = \begin{cases} \dfrac{\sum_{i=1} \hat{a}_i}{2\rho} + \underline{y}^S & \text{if } r < \dfrac{\sum_{i=1} \hat{a}_i}{4}, \\ \dfrac{r\left[2\hat{a}_1\hat{a}_2 - \sum_i \hat{a}_i[r - \hat{K}(r)]\right]}{\rho \hat{a}_1 \hat{a}_2} + \underline{y}^S & \text{if } \dfrac{\sum_{i=1} \hat{a}_i}{4} \leq r < \dfrac{\sum_i \hat{a}_i}{2}, \\ \dfrac{2r(\hat{a}_1 - r)}{\rho \hat{a}_1} + \underline{y}^S & \text{if } \dfrac{\sum_i \hat{a}_i}{2} \leq r \leq \hat{a}_1, \\ \underline{y}^S & \text{if } r > \hat{a}_1. \end{cases}$$

For $r \leq \sum_{i=1} \hat{a}_i/4$, since $\sum_{i=1} \hat{a}_i/4 \leq \hat{A}/4 \leq \hat{A}/2$, $\hat{Y}^S - \hat{Y}^B = (\sum_i \hat{a}_i - \hat{A})/2\rho + \underline{y}^S - \underline{y}^B = 0$, implying $\hat{Y}^S = \hat{Y}^B$.

For $r \in (\sum_{i=1} \hat{a}_i/4, \min\{\hat{A}/2, \hat{a}_1\}]$, since $\hat{Y}^S$ is decreasing in r while $\hat{Y}^B$ is constant, $\hat{Y}^S < \hat{Y}^B$.

For $r \in (\min\{\hat{A}/2, \hat{a}_1\}, \max\{\hat{A}/2, \hat{a}_1\}]$, there are two subcases.

1. If $\hat{A}/2 \geq \hat{a}_1$, then for $r \in (\hat{a}_1, \hat{A}/2]$,

$$\hat{Y}^B - \hat{Y}^S = \frac{\hat{A}}{2\rho} + \underline{y}^B - \underline{y}^S = \frac{\sum_i \hat{a}_i}{2\rho} > 0.$$

[24]Strictly, if $\hat{A} = 0$, then the Cauchy-Schwarz inequality implies $\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1/2}$ and $\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1/2}$ are linearly dependent. However, if so, then so are $\boldsymbol{\alpha}$ and $\boldsymbol{\eta}$, which contradicts Assumption 2.

2. If $\hat{A}/2 < \hat{a}_1$, then for $r \in [\hat{A}/2, \hat{a}_1]$,

$$\hat{Y}^B - \hat{Y}^S = \frac{2r(\hat{A}-r)}{\rho\hat{A}} - \frac{2r(\hat{a}_1 - r)}{\rho\hat{a}_1} - (\underline{y}^S - \underline{y}^B) = \frac{2(\hat{A}-\hat{a}_1)r^2}{\rho\hat{A}\hat{a}_1} - (\underline{y}^S - \underline{y}^B),$$

which is increasing in $r > 0$. Then, since $\hat{Y}^B > \hat{Y}^S$ at $r = \hat{A}/2$, $\hat{Y}^B > \hat{Y}^S$ for all $r \in (\hat{A}/2, \hat{a}_1]$.

For $r \in (\max\{\hat{A}/2, \hat{a}_1\}, \hat{A}]$, note that $\hat{Y}^B$ is decreasing and continuous in r while $\hat{Y}^S$ is constant. Since $\hat{Y}^B > \hat{Y}^S$ at $r = \max\{\hat{A}/2, \hat{a}_1\}$ and $\hat{Y}^B \leq \hat{Y}^S$ at $r = \hat{A}$, there uniquely exists $\tilde{r} \in (\max\{\hat{A}/2, \hat{a}_1\}, \hat{A}]$ such that $\hat{Y}^S \gtreqless \hat{Y}^B$ if and only if $r \gtreqless \tilde{r}$. Note that $\tilde{r} = \hat{A}$ if and only if $\underline{y}^S > \underline{y}^B$ (i.e. (12) is satisfied)

For $r > \hat{A}$, $\hat{Y}^S = \underline{y}^S \geq \underline{y}^B = \hat{Y}^B$, where the inequality holds with strict inequality if and only if (12) is satisfied.

Recall that $\boldsymbol{\alpha}_i$ and $\boldsymbol{\eta}_i$ are linearly independent for $i = 1, 2$ since $\hat{a}_i > 0$ for $i = 1, 2$. Then the statement is proven to be true by defining $\underline{r} = \sum_i \hat{a}_i/4$ and $\overline{r} = \tilde{r}$.

B.5.2 $\hat{a}_2 = 0$

Since $\hat{a}_1 > 0$, from Proposition 11, $\hat{Y}^S$ is written as

$$\hat{Y}^S = \begin{cases} \dfrac{\hat{a}_1}{2\rho} + \underline{y}^S & \text{if } r < \dfrac{\hat{a}_1}{2}, \\ \dfrac{2r(\hat{a}_1 - r)}{\rho\hat{a}_1} + \underline{y}^S & \text{if } \dfrac{\hat{a}_1}{2} \leq r < \hat{a}_1, \\ \underline{y}^S & \text{if } r \geq \hat{a}_1. \end{cases}$$

Note also that since $\hat{A} - \hat{a}_1 = \hat{A} - \sum_i \hat{a}_i = 2\rho(\underline{y}^S - \underline{y}^B)$, $\hat{A} > \hat{a}_1$ if and only if $\underline{y}^S > \underline{y}^B$. Hereafter we consider two cases in order: $\underline{y}^S > \underline{y}^B$ and $\underline{y}^S = \underline{y}^B$.

In Cases of $\underline{y}^S > \underline{y}^B$: For $r \leq \hat{a}_1/2$, since $\hat{a}_1/2 < \hat{A}/2$, $\hat{Y}^S - \hat{Y}^B = (\hat{a}_1 - \hat{A})/2\rho + \underline{y}^S - \underline{y}^B = 0$, implying $\hat{Y}^S = \hat{Y}^B$. For $r \in (\hat{a}_1/2, \hat{A}/2]$, since $\hat{Y}^S$ is decreasing in r while $\hat{Y}^B$ is constant, $\hat{Y}^S < \hat{Y}^B$. For $r > \hat{A}/2$, there are two subcases.

1. Suppose $\hat{A}/2 \geq \hat{a}_1$. For $r \in (\hat{A}/2, \hat{A}]$, $\hat{Y}^B$ is decreasing and continuous in r while $\hat{Y}^S$ is constant. Since $\hat{Y}^B > \hat{Y}^S$ at $r = \hat{A}/2$ and $\hat{Y}^B = \underline{y}^B < \underline{y}^S = \hat{Y}^S$ at $r = \hat{A}$, there uniquely exists $\tilde{r} \in (\hat{A}/2, \hat{A})$ such that $\hat{Y}^S \gtreqless \hat{Y}^B$ if and only if $r \gtreqless \tilde{r}$. For $r > \hat{A}$, $\hat{Y}^S = \underline{y}^S > \underline{y}^B = \hat{Y}^B$.

2. Suppose $\hat{A}/2 < \hat{a}_1$. For $r \in (\hat{A}/2, \hat{a}_1]$, since $\hat{a}_1 < \hat{A}$,

$$\hat{Y}^B - \hat{Y}^S = \frac{2r(\hat{A}-r)}{\rho\hat{A}} - \frac{2r(\hat{a}_1 - r)}{\rho\hat{a}_1} - (\underline{y}^S - \underline{y}^B) = \left(\frac{4r^2}{\hat{A}\hat{a}_1} - 1\right)(\underline{y}^S - \underline{y}^B),$$

which is increasing in r since $\underline{y}^S > \underline{y}^B$. Since $\hat{Y}^B > \hat{Y}^S$ at $r = \hat{A}/2$, $\hat{Y}^B > \hat{Y}^S$ for all $r \in (\hat{A}/2, \hat{a}_1]$. For $r \in (\hat{a}_1, \hat{A}]$, $\hat{Y}^B$ is continuous and decreasing in r while $\hat{Y}^S$ is constant. Since $\hat{Y}^B > \hat{Y}^S$ at $r = \hat{a}_1$ and $\hat{Y}^B < \hat{Y}^S$ at $r = \hat{A}$, there uniquely exists $\tilde{r} \in (\max\{\hat{A}/2, \hat{a}_1\}, \hat{A})$ such that $\hat{Y}^S \gtreqless \hat{Y}^B$ if and only if $r \gtreqless \tilde{r}$. For $r > \hat{A}$, $\hat{Y}^S = \underline{y}^S > \underline{y}^B = \hat{Y}^B$.[25]

Note that (12) is satisfied since $\underline{y}^S > \underline{y}^B$. Then the statment is proven to be true by defining $\underline{r} = \hat{a}_1/2$ and $\overline{r} = \tilde{r}$.

In Cases of $\underline{y}^S = \underline{y}^B$: Note the following facts: (i) (12) is violated; and (ii) $\hat{A} = \hat{a}_1$.

For $r \leq \hat{a}_1/2 = \hat{A}/2$, since $\hat{a}_1/2 \leq \hat{A}/2$, $\hat{Y}^S - \hat{Y}^B = (\hat{a}_1 - \hat{A})/2\rho + \underline{y}^S - \underline{y}^B = 0$, implying $\hat{Y}^S = \hat{Y}^B$. For $r \in (\hat{A}/2, \hat{A}] = (\hat{a}_1/2, \hat{a}_1]$, since $\hat{A} = \hat{a}_1$,

$$\hat{Y}^B - \hat{Y}^S = \frac{2r(\hat{A}-r)}{\rho\hat{A}} - \frac{2r(\hat{a}_1 - r)}{\rho\hat{a}_1} - (\underline{y}^S - \underline{y}^B) = 0.$$

For $r > \hat{A}$, $\hat{Y}^S = \underline{y}^S = \underline{y}^B = \hat{Y}^B$.

Therefore $\hat{Y}^S = \hat{Y}^B$ for all r.

B.6 Interpretation of Assumption 2

Inequalities (10) and (11) in Assumption 2 have a meaning that explicit (implicit, resp.) incentives have only a small effect on the nonverifiable (verifiable, resp.) performance measurement. By assuming such task structures, we eliminate possibilities of excess explicit incentives combined with a negative informal bonus for success of the nonverifiable signal.

Inequality (10) is required under task bundling. Recall that the incentive compatibility constraints under task bundling (14) is simplified by the first order condition as $\gamma_n e_n = \eta_n \Delta\overline{w} + \mu_n \overline{\beta}$. Then, as in Section 4.2, the marginal effort of increasing the implicit and explicit incentives on task n is μ_n/γ_n and η_n/γ_n, respectively. Since α_n, μ_n, and η_n

[25]Solving the equations yields the explicit expression of $\tilde{r}$ as

$$\tilde{r} \equiv \frac{\hat{A} + \sqrt{\hat{A}^2 - 2\rho\hat{A}(\underline{y}^S - \underline{y}^B)}}{2}.$$

are the marginal benefit, the marginal success probability of the nonverifiable signal, and the marginal success probability of the verifiable signal of effort e_n, we have the following interpretations:

- $\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'$: the marginal benefit of the implicit incentive;
- $\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'$: the marginal success probability of the verifiable signal of the implicit incentive;
- $\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'$: the marginal benefit of the explicit incentive; and
- $\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'$: the marginal success probability of the verifiable signal of the explicit incentive.

Then the fraction $\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'/\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\mu}'$ expresses the ratio of the effects of an incremental increase of the implicit incentive on the principal's benefit and the success probability of the verifiable signal. Similarly, the fraction $\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'/\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1}\boldsymbol{\eta}'$ expresses the ratio of the effects of an incremental increase of the explicit incentive. Inequality (10) in turn implies that an incremental increase of the informal bonus is effective on the principal's benefit relative to an incremental increase of the explicit incentive while an incremental increase of explicit incentive is relatively effective on the verifiable signal. Inequality (11) has the same implication on the signal structure under task separation.

Inequalities (10) and (11) are also interpreted in a geometric fashion. From the cosine formula, (10) is equivalent to

$$\cos\theta(\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1/2}, \boldsymbol{\mu}\boldsymbol{\Gamma}^{-1/2}) \geq \cos\theta(\boldsymbol{\alpha}\boldsymbol{\Gamma}^{-1/2}, \boldsymbol{\eta}\boldsymbol{\Gamma}^{-1/2}) \cos\theta(\boldsymbol{\eta}\boldsymbol{\Gamma}^{-1/2}, \boldsymbol{\mu}\boldsymbol{\Gamma}^{-1/2}),$$

where $\theta(\cdot,\cdot)$ is the angle between the vectors. Note that the vectors $\boldsymbol{\alpha}$, $\boldsymbol{\mu}$, and $\boldsymbol{\eta}$ express the marginal effects of effort on the benefit and the success probability of the nonverifiable and verifiable performance measurements, respectively. Inequality (10) requires the condition of alignments of the vectors with making an appropriate adjustment based on the cost parameter γ_n.[26] Specifically, the inequality has two implications: (i) $\boldsymbol{\alpha}$ and $\boldsymbol{\mu}$ are relatively aligned; and (ii) $\boldsymbol{\eta}$ must be less aligned to at least either $\boldsymbol{\alpha}$ or $\boldsymbol{\mu}$.

[26]Schöttner (2008) similarly points out that when the degree of multitasking incentive problem is measured by alignment of the vectors of the marginal effects, the adjustment based on the cost difference should be taken into account.

C Appendix: Generalization of the Cost Functions

This section provides a condition under which all the results in this article qualitatively holds even if the cost function is generalized. Recall that we have assumed that the cost function is additively separable in cost of each task. We now generalize the cost function in the following way. Without loss of generality, let $\mathcal{N}_1 = \{1, \ldots, N_1\}$ and $\mathcal{N}_2 = \{N_1 + 1, \ldots, N\}$. Then define the following matrices that are symmetric and positive definite:

$$\Gamma_1 \equiv \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1N_1} \\ \vdots & \ddots & \vdots \\ \gamma_{N_1 1} & \cdots & \gamma_{N_1 N_1} \end{bmatrix}, \quad \Gamma_2 \equiv \begin{bmatrix} \gamma_{N_1+1,N_1+1} & \cdots & \gamma_{NN_1} \\ \vdots & \ddots & \vdots \\ \gamma_{N_1 N_1+1} & \cdots & \gamma_{NN} \end{bmatrix},$$

where $\gamma_{ii} > 0$ and $\gamma_{ij} = \gamma_{ji}$ for all $i, j = 1, \ldots, N$, and let

$$\Gamma \equiv \begin{bmatrix} \Gamma_1 & \mathbf{0} \\ \mathbf{0} & \Gamma_2 \end{bmatrix}.$$

Then under task bundling, agent 1's cost is defined as $\boldsymbol{e}\boldsymbol{\Gamma}\boldsymbol{e}'/2$ and under task separation, agent i's cost is defined as $\boldsymbol{e}_i\boldsymbol{\Gamma}_i\boldsymbol{e}_i'/2$.

All the results derived in the analysis are valid as long as Γ_i satisfies the above property. Recall that the case of additive separable costs imposes the assumption that $\gamma_{ij} = 0$ for all $i = 1, \ldots, N$ and $j \neq i$. Even if the cost is not additive separable, given that Γ_i is symmetric and positive definite for $i = 1, 2$, so is Γ and then Γ have the inverse matrix Γ^{-1}. Based on this Γ^{-1}, we can construct the parameters such as A, M, a_i, and m_i and apply all the provided proof.

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